Format for experiment reports(1)

1. Title (제목)

2. Purpose (목적)
   - Clear and short statement identifying the purpose of the experiment and the report
   - Ex) The purpose of this experiment is to test the diminished intensity of light as it spreads unhindered in all directions.

3. Background theories or concepts (배경 이론)
   - Describing important theories, ideas, concepts, formulae relating to the experiment based on your understanding
   - It contains the information that an uninformed reader will find useful to understand and interpret the rest of the report.
   - Do not make a copy of what you find in the references
   - Make your own explanation of the experiment

Format for experiment reports(2)

4. Equipment and apparatus for the experiment
    - Including your own sketch and arrangement of experimental setup and connection
    - It might be a rough sketch, but should include essential pieces of equipment
    - Describe components of instrument for the purpose, the range of measurement, the precision, etc.

5. Procedure
    - Steps used to do the experiment
    - Details should be sufficient so that the reader may repeat the experiment and verify your experimental results.
    - Referring the reader to an apparatus diagram can often significantly reduce the need for textual descriptions.

Format for experiment reports(3)

6. Results and analysis
    - Including tables and graphs
    - Do not forget to put captions in tables and graphs
    - There should be axis labels with units in all graphs.
    - Be careful with significant numbers and the precision in the data.
    - All numbers should appear with units.

7. Discussion
    - Make comparison with theoretical expectation or known-values
    - Discuss the accuracy and precision of your measurements
    - Systematic error analysis
    - Sources of error, problems in experimental setup or procedure, proposal for the better method to do the experiment, discussion with other members of your group…

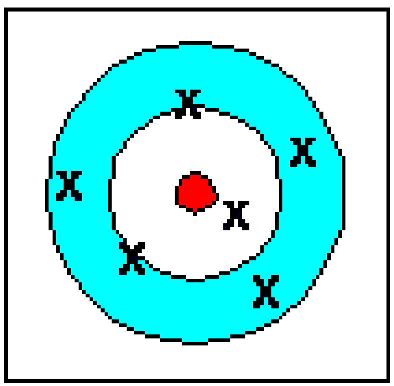
Format for experiment reports(4)

8. Conclusion
   - Statement of each of the generalizations that you are able to draw on the basis of your analysis of data
   - Short summary of experiment and discussion, significance of results
   - Such conclusions should pertain to the purpose stated in the report.
   - Any equations or proportions which have been developed from the experiment.

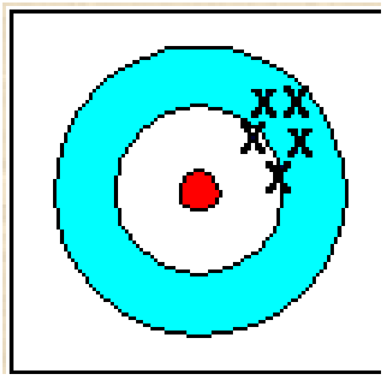
9. References
   - Referenced to the source of information, including author, title, publisher, copyright date, and page, or equivalent information so that a reader could find and read the information from the source.

Accuracy vs. Precision

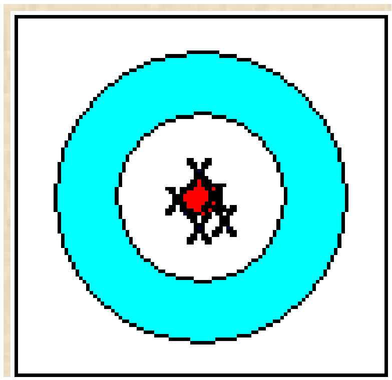
- Accuracy : refers to how closely a measured value agrees with the correct value. (measure of correctness)
- Precision : refers to how closely individual measurements agree with each other, whether it is accurate or not – number of significant figures, measurement instruments



Accurate and not precise



Not accurate and precise



Accurate and precise

- In any measurement, the number of significant figures is critical. The number of significant figures is the number of digits believed to be correct by the person doing the measuring. It includes one estimated digit. So, the concept of significant figures deal with precision.

Determining the number of significant figures

- All non-zero digits are significant. Example: '123.45' has five significant figures: 1,2,3,4 and 5.
- Zeros appearing in between two non-zero digits are significant. Example: '101.12' has five significant figures: 1,0,1,1,2.
- All zeros appearing to the right of an understood decimal point and non-zero digits are significant. Example: '12.2300' has six significant figures: 1,2,2,3,0 and 0. The number '0.00122300' still only has six significant figures (the zeros before the '1' are not significant).
- All zeros appearing in a number without a decimal point and to the right of the last non-zero digit are not significant unless indicated by a bar. Example: '1300' has two significant figures: 1 and 3. The zeros are not considered significant because they don't have a bar. However, 1300.0 has five significant figures.
- How many significant figures ?

976.4 84000 0.0094 301.07

4.000 10 400. $4.00*10^2$

Rules for working with significant figures

- Addition and subtraction with significant figures
  - When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.

- Multiplication and division with significant figures
  - When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.

$$\begin{array}{r} 4.783\underline{2} \\ 1.23\underline{4} \\ +\ 2.0\underline{2} \\ \hline 8.0\underline{372} \\ \Downarrow \text{rounding} \\ 8.04 \end{array} \qquad \begin{array}{r} 1.023\underline{6} \\ -\ 0.9726\underline{8} \\ \hline 0.050\underline{92} \\ \Downarrow \text{rounding} \\ 0.0509 \end{array} \qquad \begin{array}{r} 2.872\underline{3} \\ \times\ \ 1.\underline{6} \\ \hline 4.\underline{59568} \\ \Downarrow \text{rounding} \\ 4.6 \end{array} \qquad \begin{array}{r} 45.\underline{2} \\ \div\ 6.357\underline{8} \\ \hline 7.1\underline{093775} \\ \Downarrow \text{rounding} \\ 7.11 \end{array}$$

Errors and data analysis

- Statistical error:
  - **Caused by random (and therefore inherently unpredictable) fluctuations in the measurement apparatus**
  - **Error that is reduced with more measurements**
- Systematic error:
  - **Caused by an unknown but nonrandom fluctuation. If the cause of the systematic error can be identified, then it can usually be eliminated. Such errors can also be referred to as uncertainties**
  - **Biases in measurement which lead to measured values being systematically too high or too low**
  - **A systematic error is any biasing effect, in the environment, methods of observation or instruments used, which introduces error into an experiment and is such that it always affects the results of an experiment in the same direction**
  - **Error that has nothing to do with the number of measurements.**
  - **Making reasonable analysis of the systematic error of your measurement is very important part of report**

Statistical error

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Mean value

$$\sigma \cong s \equiv \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Standard deviation of data

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$

Standard deviation of mean value

With N times measurements(data: x_1, x_2, …. x_N), estimated real value is (50% confidence interval)

$$Value = \bar{x} \pm 0.6745\frac{\sigma}{\sqrt{N}} = \frac{1}{N}\sum_{i=1}^{N} x_i \pm 0.6745\sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$

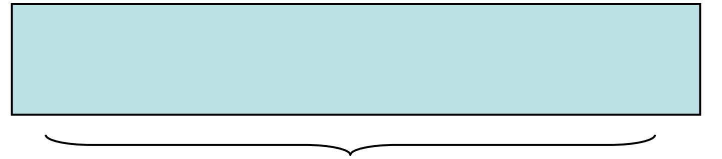
Probable error: the amount by which the mean value of a sample is expected to vary because of chance alone

Error propagation

x = 8.1 $\pm$ 0.1

y = 2.34 $\pm$ 0.04

Error in x+y = ?



x = 8.1 $\pm$ 0.1

y = 1.3456 $\pm$ 0.0001

Error in x*y = ?

$$\bar{z} = f(\bar{x}, \bar{y}, \cdots)$$

- $\bar{z}$ **:computed value**

- $\bar{x}$ $\bar{y}\cdots$ **:measured value**

$$\sigma_z^{\ 2} = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^{\ 2} + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^{\ 2} + \cdots$$

모집단, 표본

- 모집단(population): 측정 횟수를 무한번 했을때 얻는 값
  (관심의 대상이 되는 전체 집단)
- 표본(sample): N번의 측정값
  (모집단에서 조사 대상으로 채택된 일부 집단)

	평균	편차	표준편차
모집단의 특성(모수)	$\mu = \lim_{N\to\infty}\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$	$x_i - \mu$	$\sigma = \sqrt{\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}$
표본의 특성 (통계량)	$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$	$x_i - \bar{x}$	$s = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$

모평균, 모표준편차의 추정

- 모평균의 추정

$$\mu \cong \bar{x}$$

– 표본평균은 편차의 제곱의 합을 최소로 하는 것이다.(최소 제곱법)

$$\sum_{i=1}^{N}(x_i - \bar{x})^2 = \sum_{i=1}^{N}(x_i)^2 - 2\bar{x}\sum_{i=1}^{N}x_i + N\bar{x}^2 \quad \Rightarrow \quad \bar{x} = \frac{1}{N}\sum_{i=1}^{N}x_i$$

- 모표준편차의 추정

$$\sigma \cong s \equiv \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

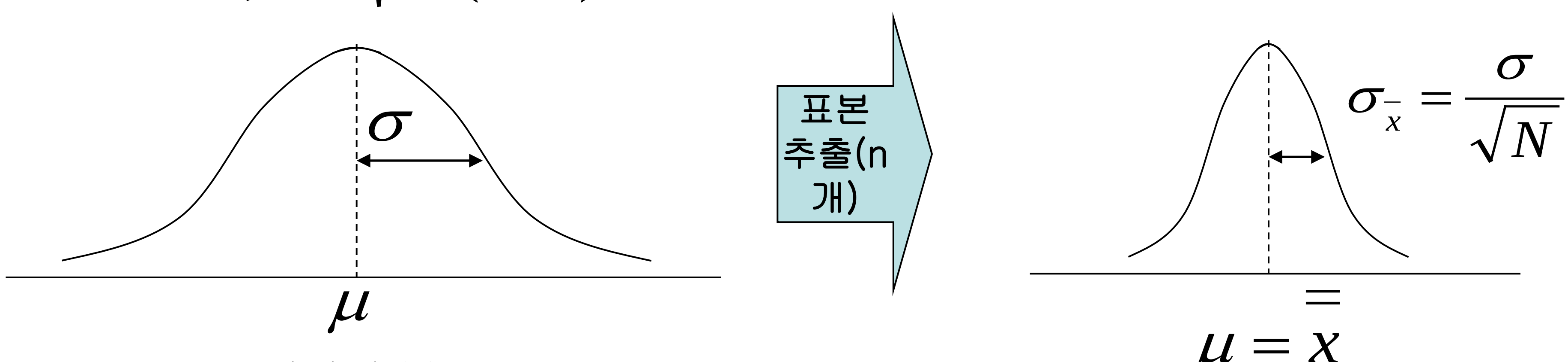
모평균 추정에 수반되는 오차

$$\mu = \bar{\bar{x}} = \frac{1}{N}\sum_{i=1}^{N}\overline{x_i}$$:표본 평균의 평균

오차의 전파를 이용

$$\sigma_{\bar{x}}^{\ 2} = \sum_{i=1}^{N}\left(\frac{\partial \mu}{\partial x_i}\right)^2 \sigma_{x_i}^{\ 2} = \sum_{i=1}^{N}\left(\frac{1}{N}\right)^2 \sigma^2 = \frac{\sigma^2}{N}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$:표본 평균의 표준편차 (σ: 모표준편차)



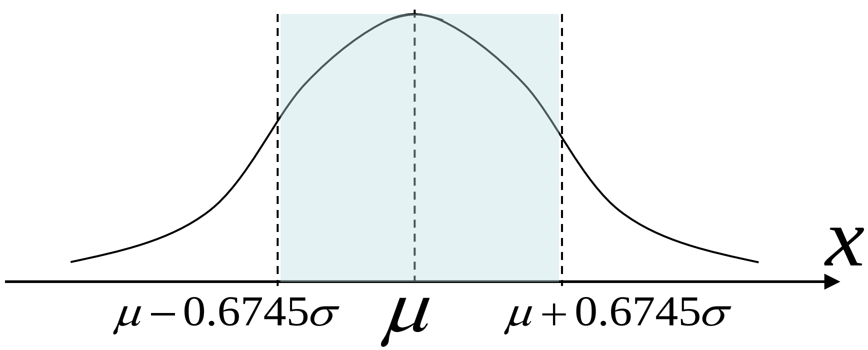
모집단의 분포

표본 평균의 분포

정규분포

(Gaussian or Normal Distribution)

$$P(x,\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\right)$$



$$\int_{-\infty}^{\infty}P(x,\mu,\sigma)dx=1$$

$$Mean=\int_{-\infty}^{\infty}xP(x,\mu,\sigma)dx=\mu$$

$$S.D.^2=\int_{-\infty}^{\infty}(x-\mu)^2P(x,\mu,\sigma)dx=\sigma^2$$

$$\int_{\mu-0.6745\sigma}^{\mu+0.6745\sigma}P(x,\mu,\sigma)dx=\frac{1}{2}$$

실험 데이터의 표현방식

N회 측정하여 $x_1, x_2, \ldots. x_N$ 의 데이터를 얻은 경우 50% 의 신뢰도로 참값을 추정하면 다음과 같다.

$$Value = \bar{x} \pm 0.6745\frac{\sigma}{\sqrt{N}} = \frac{1}{N}\sum_{i=1}^{N} x_i \pm 0.6745\sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$

확률오차: 이 값을 경계로 더 큰 오차와 작은 오차가 일어날 확률이 같게 되는 것으로 정의함