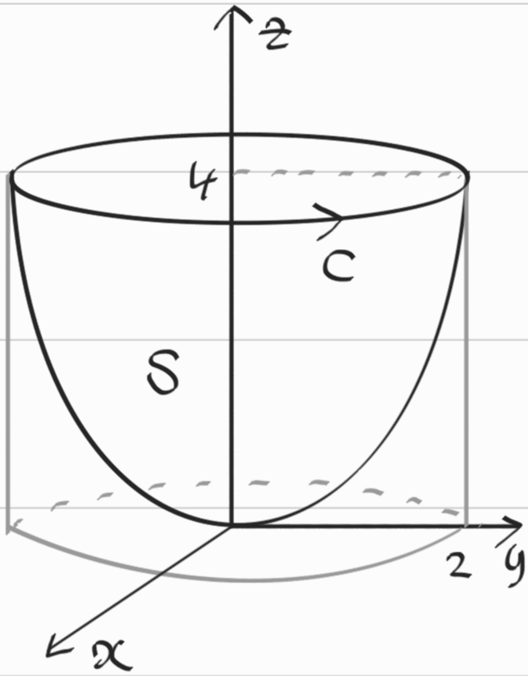
# 공업수학

#1. $\vec{F} = \langle x^2z^2, \ y^2z^2, \ xyz \rangle$




$$\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$$

$C : \ x^2 + y^2 = 4$

$$= \int_C \vec{F} \cdot d\vec{r}$$

$\vec{r}(t) = \langle 2\cos t, 2\sin t, 4 \rangle$

$0 \le t \le 2\pi$

$$= \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) \, dt$$

$$= \int_0^{2\pi} \langle 64\cos^2 t, \ 64\sin^2 t, \ 16\sin t\cos t \rangle$$

$$\cdot \langle -2\sin t, \ 2\cos t, \ 0 \rangle \, dt$$

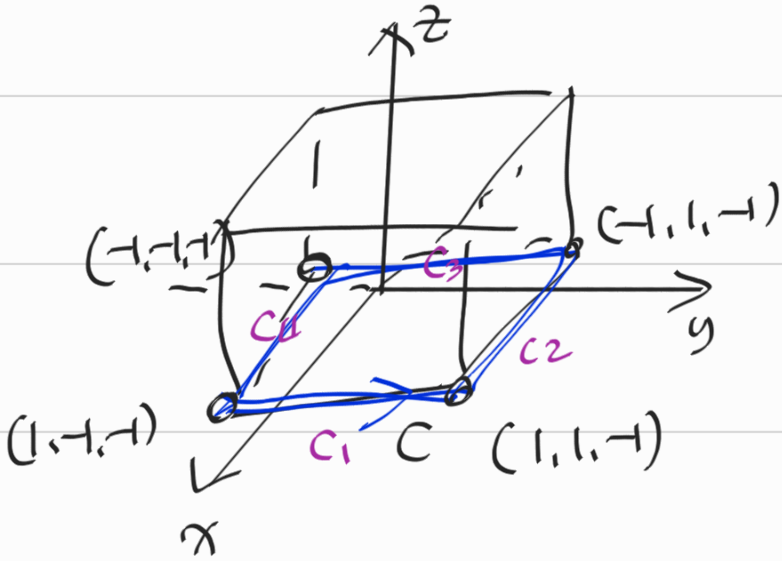
$$= 128\int_0^{2\pi} -\cos^2 t\sin t + \sin^2 t\cos t \ dt$$

$$= 128 \left[ \frac{1}{3}\cos^3 t + \frac{1}{3}\sin^3 t \right]_0^{2\pi}$$

$$= 0$$

#3. $\vec{F} = \langle xyz,\ xy,\ x^2yz \rangle$

$$\iint_S \text{curl}\,\vec{F} \cdot d\vec{S}$$




$$= \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} + \int_{C_3} \vec{F} \cdot d\vec{r} + \int_{C_4} \vec{F} \cdot d\vec{r}$$

$$= \int_{-1}^{1} \langle -t, t, -t \rangle \cdot \langle 0, 1, 0 \rangle \, dt$$

$$+ \int_{-1}^{1} \langle t, -t, -t^2 \rangle \cdot \langle -1, 0, 0 \rangle \, dt$$

$$+ \int_{-1}^{1} \langle -t, t, t \rangle \cdot \langle 0, -1, 0 \rangle \, dt$$

$$+ \int_{-1}^{1} \langle t, -t, t^2 \rangle \cdot \langle 1, 0, 0 \rangle \, dt$$

$$= \int^{1} t - t - t + t \, dt = 0$$

$\int_{-1} t - t - t + t \; dt$

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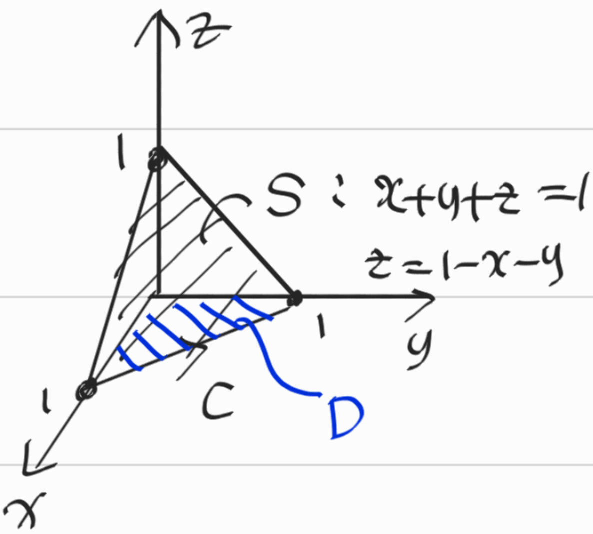
$C_1 : \quad \vec{r}_1(t) = \langle 1, t, -1 \rangle, \quad -1 \le t \le 1$

$C_2 : \quad \vec{r}_2(t) = \langle -t, 1, -1 \rangle, \quad -1 \le t \le 1$

$C_3 : \quad \vec{r}_3(t) = \langle -1, -t, -1 \rangle, \quad -1 \le t \le 1$

$C_4 : \quad \vec{r}_4(t) = \langle t, -1, -1 \rangle, \quad -1 \le t \le 1$

#5. $\vec{F} = \langle x+y^2, \; y+z^2, \; z+x^2 \rangle$




$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y^2 & y+z^2 & z+x^2 \end{vmatrix}$$

$$= \langle -2z, \; -2x, \; -2y \rangle$$

$\int_C \vec{F} \cdot d\vec{r}$

$= \iint_S \text{curl}\,\vec{F} \cdot d\vec{S}$

$\vec{r}(u,v) = \langle u, v, 1-u-v \rangle$

$\vec{r}_u = \langle 1, 0, -1 \rangle$

$\vec{r}_v = \langle 0, 1, -1 \rangle$

$$= \iint_S \langle -2z, -2x, -2y \rangle \cdot d\vec{S}$$

$$\vec{r}_u \times \vec{r}_v = \langle 1, 1, 1 \rangle$$

$$= \iint_D \langle -2(1-u-v), -2u, -2v \rangle \cdot \langle 1, 1, 1 \rangle \, dA$$

$$= \iint_D -2 \, dA$$

$= -2 \times$ D의 넓이

$= -1.$