공업수학

#2.(3) $(1+i)^i = e^{i\ln(1+i)}$

$\ln(1+i) = \mathrm{Log}|1+i| + i\arg(1+i)$

$= \mathrm{Log}\sqrt{2} + i\left(\frac{\pi}{4} + 2n\pi\right)$

$\Rightarrow e^{i\ln(1+i)} = e^{i(\mathrm{Log}\sqrt{2} + i(\frac{\pi}{4}+2n\pi))}$

$= e^{-(\frac{\pi}{4}+2n\pi) + i\mathrm{Log}\sqrt{2}}$

$= e^{-(\frac{\pi}{4}+2n\pi)}\left(\cos(\mathrm{Log}\sqrt{2}) + i\sin(\mathrm{Log}\sqrt{2})\right)$

$\therefore$ $(1+i)^i$ 의 주치 $= e^{-\frac{\pi}{4}}\left(\cos(\mathrm{Log}\sqrt{2}) + i\sin(\mathrm{Log}\sqrt{2})\right)$

#3. (5) $\sinh z = i$

$\Rightarrow \dfrac{e^z - e^{-z}}{2} = i$

$\Rightarrow e^z - e^{-z} = 2i$

$\Rightarrow e^{2z} - 2ie^z - 1 = 0$

$$\Rightarrow \quad (e^z - i)^2 = 0$$

$$\Rightarrow \quad e^z = i$$

$$\Rightarrow \quad z = \ln i = \operatorname{Log}|i| + i \arg(i)$$

$$= \operatorname{Log} 1 + i\left(\frac{\pi}{2} + 2n\pi\right)$$

$$= i\left(\frac{\pi}{2} + 2n\pi\right)$$

$$\therefore \quad z = i\left(\frac{\pi}{2} + 2n\pi\right) \qquad n \in \mathbb{Z}.$$

(7) $\sin z = \cos z$

$$\Rightarrow \quad \frac{e^{iz} - e^{-iz}}{2i} = \frac{e^{iz} + e^{-iz}}{2}$$

$$\Rightarrow \quad e^{iz} - e^{-iz} = i\left(e^{iz} + e^{-iz}\right)$$

$$\Rightarrow \quad e^{2iz} - 1 = i\left(e^{2iz} + 1\right)$$

$$\Rightarrow \quad (1-i)e^{2iz} = 1+i$$

$$\Rightarrow \quad e^{2iz} = \frac{1+i}{1-i} = i$$

$$\Rightarrow \quad \underbrace{2iz}_{} = \ln i = \underbrace{i\left(\frac{\pi}{2}+2n\pi\right)}_{}$$

$$\Rightarrow \quad z = \frac{\pi}{4} + n\pi, \qquad n \in \mathbb{Z}.$$