A STUDY ON THE COMPETITIVE SIGNALLING IN LABOR MARKET

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CONTENT

Ⅰ. Introduction

One of the most remarkable achievements in the economics of information is the development of the concept of signalling, which was formally proposed by Spence (1973). Recently, two types of signalling models have been developed. One is the typical Spencian model(Model Ⅰ) where workers(informed agents) move first by choosing signals before firms offer wages for each signal. One of the major problems in this type of models is the multiplicity of the Nash equilibria. Kreps(1985) characterized the set of sequential equilibria in the Spencian model, and further refined the sequential equilibria by the so-called weak dominance criterion. The other type of signalling model is the one(Model Ⅱ) where firms(uninformed agents) move first by competitively offering their contracts before workers choose signals. Riley(1975, 1979) has introduced competition among firms for contracts in the Spencian labor market to resolve the multiplicity problem. The idea behind this approach is that the competition among firms with respect to contracts to offer would eliminate all but the Pareto-dominating contracts.

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In this paper, we try a different approach. Here the strategy space for firms is extended such that a firm conditions its wage offers on distributions of signals as well as on the absolute levels of signals. And we consider the two types of signalling models with this extended strategy space. First, each firm can protect the profitability of its contract by conditioning its wage offers on the signal distributions(particular, by announcing the rank-order contract), because each signal distribution conveys information about the productivity-signal relationship. Second, a set of contracts adopted by firms will induce workers to compete with each other in choosing their signals.

The main purpose of this paper is to show how these characteristics associated with the extended strategy space will affect the existence or the uniqueness of an equlibrium in each of the two models. In particular, we establish that these characteristics lead to the unique equilibrium in each model, which resolves the multiplicity problem in the Spencian model and the non-existence problem in Riley or Rothschild-Stiglitz(R-R-S). And it is shown that the unique equilibrium is characterized by the fact that each type of a worker is completely separated from the others in the most efficient way. We also showed that the unique equilibrium is supported by the rank-order contract which offers a worker wage by the ranking of his signal, not by its absolute level.

In the following section, a basic framework is presented. In section Ⅲ and Ⅳ it is shown that there exists a unique equilibrium in each of the two models, which leads to a complete separation among different types of workers. And the optimality of this equilibrium is examined together with some concluding remarks in section V.

Ⅱ. A Basic Framework

Consider a competitive labor market where workers have private information about their types. There are a continuum of firms and a continuum of types of workers. Each types of worker differs from the others in productivity, z, where z belongs to a closed interval $[z_1, z_2]$, where $z_1 < z_2$. Firms are assumed to know the distribution of types $F(z)$, where $F(z)$ is a cumulative distribution function of z. $F(z)$ is assumed to be strictly increasing and differentiable for all z. So the proportion of population of one type to the whole population of workers is negligible.

Suppose there is some exogenous activity workers engage in to signal their types before they enter the market. For this activity to be an effective signal, it must be more costly for the less productive workers than for the more productive workers. Then the wage offers by firms to the workers will depend on the signals they have chosen. We assume that the signalling activity does not affect productivity of each type of worker, which is not essential for the results in this model. The utility of worker whose productivity is z, when he gets wage w by choosing a signal s, is assumed to be

$$U(w,s\ ;\ z) = w - c(z)s$$

where $c(z)$ is the unit cost of signal for a worker of z-type. So the signalling technology is assumed to be noiseless and constant returns to scale. The following assumption is essential in this model :

$$c'(z) < 0 \tag{1}$$

Equation (1) implies that signalling is more costly for the less able worker than for the more able workers.

The production technology is assumed to be constant returns to scale so that profit per employee is simply productivity less wage. This production technology also implies that a firm can hire as many workers as it wants.

Each firm offers its contract that specifies how its wage offers to workers will be based on signals. Let $g(s)$ (or g) be a distribution of signals chosen by j to a worker will depend on the level of his signal s and the signal distribution g :

$$W^j = W^j(s\ ;\ g) \tag{2}$$

$W^j(s\ ;\ g')$ is a wage function (or a wage-signal schedule) specifying the wage offers for different signals given a certain signal distribution g'. So a wage contract $W^j(\cdot)$ maps from $R \times M$ to R, where(M, d_M) is a metric space with a metric

$$d_M(g_1, g_2) = \operatorname*{Sup}_{s} \| g_1(s) - g_2(s) \| \tag{3}$$

Let $W(s ; g)$ be a market wage contract which is an upper envelope of a set of individual wage contracts $\{W^1(s ; g), W^2(s ; g), \cdots\cdots\}$. That is, a market wage contract $W(s ; g)$ describes the highest wage offer that is announced by firms for each signal and each signal distribution. Then given $W(s : g)$, we can define a market wage function $W(s ; g_1)$ for any particular signal distribution g_1. When a wage contract is based on distributions of signals as well, each worker cares about the signals that the other workers are going to choose. In this sense a market wage contract induces competition among workers. Now we will describe how workers compete with each other in choosing their signals given a market contract.

Before they choose their signals, all workers are assumed to share the common conjecture $W^e(\quad)$ about the future market wage contract and the common conjecture g^e about the distribution of signals that is going to be realized. These conjectures W^e, g^e will determine the expected wage function $W^e(s ; g^e)$, given which each type of worker chooses his contracts before workers choose signals, the conjecture W^e will be just the actual market wage contract W.

Since there is a continuum of types of workers, an individual's choice of a signal will not affect the distribution of signals. Knowing this fact, each worker takes the expected market wage function $W^e(s : g^e)$ as given when he chooses his signal. Thus

$$S(z ; W^e(s ; g^e)) = \operatorname*{Argmax}_{s} W^e(s ; g^e) - c(z)s$$ [1)]

We define an optimal response $G(W^e(s ; g(($ of workers to $W^e(s ; g^e)$ as a distribution of signals generated by given the expected market wage function $W^e(s ; g^e)$. and a set of signals chosen by workers given an expected wage function $W^e(s ; g^e)$ is going to be denoted by $T(W^e(s ; g^e))$. An actual wage function $W_a(s ; g^e)$ is the wage function $W(s ; G(W^e(s ; g^e)))$. which workers are actually faced with given their optimal response $G(W^e(s ; g^e))$. And a realized wage function $W_r(s ; g^e)$ is the actual wage function $W_a(s ; g^e) \mid T(W^e(s ; {}^e g^e))$ restricted to $T(W^e(s ; g^e))$. It should be noted that the expected wage function $W^e(s\,;\,g^e)$ may not be equal to the actual wage function $W_a(s\,;\,g^e)$. In that case, some workers may regret their choices of signals. A self-fulfilling wage function $W^*(s\,;\,g^e)$ is the actual wage function $W_a(s\,;\,g^e)$ such that

$$W_a(s\,;\,g^e) = W^e(s\,;\,g^e)$$

Also a realized self-fulfilling wage function $W_r^*(s\,;\,g^e)$ is the self-fulfilling wage function $W^*(s\,;\,g^e) \mid T(W^*(s\,;\,g^e))$ restricted to $T^*(W^*(s\,;\,g^e))$. Now we will discuss a property of an optimal response $G(W(s\,;\,g^e))$ of workers, which is crucial in each of the two models I and Ⅱ.

Proposition 1

For any expected wage function $W^e(s\,;\,g^e)$ an optimal response by workers $G(W^e(s\,;\,g^e))$ implies that

$$S(z^h\,;\,W^e(s\,;\,g^e)) \geq S(z^l\,;\,W^e(s\,;\,g^e)) \text{ for } z^h > z^l$$

⟨proof⟩

Let us pick up any two levels of signals from $G(W^e(s\,;\,g^e))$, s^h and s^l, where $s^h = S(z^h\,;\,W^e(s\,;\,g^e))$ and $s^l = S(z^l\,;\,W^e(s\,;\,g^e))$. Suppose $s^h < s^l$, then $W^e(s^h\,;\,g^e) < W^e(s^l\,;\,g^e)$, because otherwise s^l would not be chosen. And the following should hold by the definition of optimal response by workers who take the common conjecture g^e as fixed ;

$$W^e(s^h\,;\,g^e) - c(z^h)s^h \geq W^e(s^l\,;\,g^e) - c(z^h)s^l$$
$$W^e(s^h\,;\,g^e) - c(z^l)s^h \leq W^e(s^l\,;\,g^e) - c(z^l)s^l$$

Therefore we have

$$c(z^h)(s^l - s^h) \geq W^e(s^l\,;\,g^e) - W^e(s^h\,;\,g^e)$$
$$c(z^l)(s^l - s^h) \leq W^e(s^l\,;\,g^e) - W^e(s^h\,;\,g^e)$$

This implies that $c(z^h) \geq c(z^l)$, which contradicts our assumption (1). Q.E.D.

Proposition 1 says that the optimal signal $S(z^h\,;\,W^e(s\,;\,g^e))$ for a worker of z-type is nondecreasing in z given any expected wage function $W^e(s\,;\,g^e)$. So the relation between signals and productivities is weakly monotonic for any distribution of signals so long as the signal distribution is generated by the optimal choices of signals by all workers given any expected wage function $W^e(s\,;\,g^e)$. Then one can determine the expected productivity of a worker by matching the relative position of his signal in the distribution of signals g(s) with that of a type in the distribution of types F(z) (Matching Process). For example, if his signal is in the 45 percentile in the whole distribution of signals, his productivity is also expected to be in the 45 percentile in the distribution of types of workers. And if his signal is in between 45 and 50 percentile, so is expected to be his productivity in the distribution of types. By Proposition 1 this expectation is correct, provided that the workers make an optimal response to any expected market wage function.

Now let us describe the Matching Process more formally. Define $z^1(s')$ and $z^2(s')$ for a certain signal s′ such that

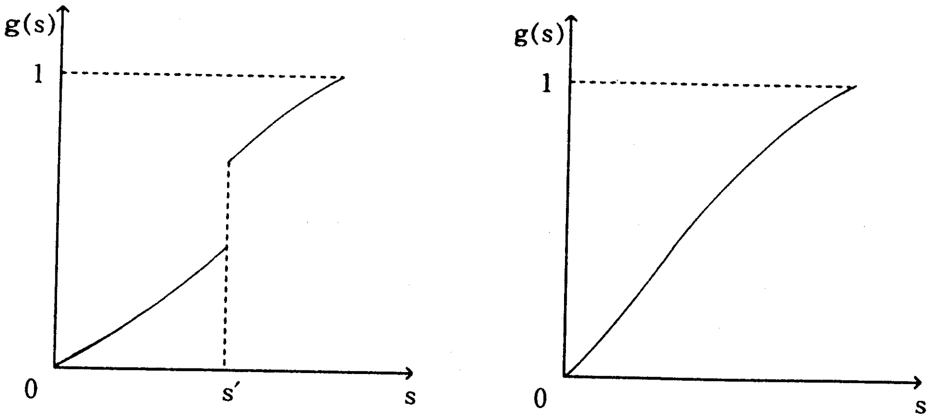
$$
\begin{aligned}
&F(z^1(s')) = g(s') \\
&F(z^2(s')) = \lim_{S \to S} g(s') \qquad \text{when } s' > 0 \\
&z^2(0) = z^1 \qquad\qquad (4)
\end{aligned}
$$

Then we can determine the expected productivity $z(s'\,;\,g)$ of a worker who has chosen s′ given a distribution of signals g as follows ;

$$
z(s'\,;\,g) = \int_{z^2}^{z^1} zf(z)dz/\{F(z^1(s')) - F(z^2(s'))\} \text{ when } z^1(s') > z^2(s') \qquad (5a)
$$

$$
= z^1(s) \text{ when } z^1(s') = z^2(s') \qquad (5b)
$$

Figure 1



(a) (b)

Equation (4) and (5) describe the Matching Process. Whenever different types of workers choose different signals(a separating optimal response), the corresponding distribution of signals g(s) will be differentiable for all s as in Figure 1(b) because F (z) is differentiable for all $z \in [z_1, z_2]$. Then the expected productivity z(s ; g) will be determined by (5b), and z(s ; g) will be also differentiable in s for all $s \in [S(z_1 ; W^e), S(z_2 ; W^e)]$, where $S(z_2 ; W^e)$ is an optimal signal for a z-type worker given an expected wage function W^e.

And whenever a set of different types of workers chooses the same signal s′ (a pooling optimal response), the signal distribution function g(s) will be discontinuous at s′ is shown in Figure 1(a). Then their expected productivity z(s′ ; g) will be determined as the average productivity of workers between $z^1(s' ; g)$ and $z^2(s' ; g)$(by (5a)). And z(s ; g) will be discontinuous at s′. Since g(s) is discontinuous at a finite number of signals at most, we can say that g(s) and z(s ; g) are discontinuous and differentiable almost everywhere.

Let us consider the following contract $W^R(s ; g)$;

$$W^R(s ; g) = z(s ; g) \text{ for all s and g} \qquad (6)$$

We will call this rank-order contract because it pays wage by the relative position (or rank-order) of a signal in a distribution of signals. Since the rank-order contrac $W^R(s\,;\,g)$ satisfies (6) for all signals given any signal distribution, it is always consis tent with the information revealed by a distribution of signals that is generated by th optimal response of workers as described above.

Ⅲ. The Model Ⅰ

Since firms do not offer their contracts before workers choose their signals in mode Ⅰ, workers will have a conjecture W^e about the future market contract nd a conje ture g^e about the signal distribution to be realized when they choose their signal: Then a Nash equilibrium requires that each firm should offer an optimal contract wit respect to the distribution g of signals, which is defined as follows.

A contract $W(s\,;\,g)$ is optimal with respect to a distribution g of signals if, give the distribution g of signals, there does not exist a contract that can make positiv profits.

Then, by the competition among firms with respect to wage offers given g, the opt mal contract will offer each realized signal the wage as much as the expected produ tivity $z(s\,;\,g)$ derived by (5). That is,

$$\begin{aligned} W^*(s\,;\,g_1) &= z(s\,;\,g_1) \text{ for all } s \in T(W^*(s\,;\,g_1)) \\ &= 0 \text{ for all } s \notin T(W^*(s\,;\,g_1)) \end{aligned} \qquad (7a)$$

In fact, the wage offer for the signal that does not belong to $T(W(s\,;\,g))$ can b anything so long as it does not affect g or $T(W(s\,;\,g))$. Since the Nash equilibriu requires that each worker should be satisfied with his choice of signal ex-post, should be defined as follows.

A Nash equilibrium is a self-fulfilling wage function of an optimal contract W(s g) with respect to g_1, where

$$g_1 = G(W(s\,;\,g^1)) \qquad (7b)$$

So a Nash equilibrium in this model is any wage function $W(s\,;\,g)$ that satisfies (7a) and (7b). Let us denote a wage function satisfying (7a) and (7b) by $w(s)$. Then a Nash equilibrium $w(s)$ is a self-fulfilling wage function of a wage contract such that once each type of a worker chooses his optimal signal given $w(s)$, $w(s')$ will be exactly equal to the average productivity of workers choosing a certain signal s' [2] Thus this wage function $w(s)$ satisfies the self-fulfilling property and the zero profit condition, which constitute a Spencian equilibrium. In terms of Spencian model, the conditions (7) imply that once firms have a belief $z(s\,;\,G(w(s)))\mid T(w(s))$ about the productivity-signal relationship and offer the competitive wages $w(s)$ for each signal given that belief, the optimal response of workers to the wage function $w(s)$ will confirm the belief $z(s\,;\,G(w(s))\mid T(w(s)$ and firms make zero profit for each employment of a worker. As Spence mentioned, there are many Spencian equilibria(or Nash equilibria) in general. Now we will briefly characterize the Spencian equilibria before refining them.

In general, there are two kinds of the wage functions $w(s)$. The one is called a seqarating Spencian wage function(SSWF) $w^S(s)$, given which each type of a worker chooses a different signal from those chosen by the others. In particular, $w^S(s\,;\,s_1)$ denotes a SSWF such that the lowest type of a worker chooses s_1 and is paid his productivity z_1. And the other is called a pooling Spencian wage function(PSWF) $w^P(s)$, given which a certain set of different types of workers choose the same signal.

Spence formally characterizes SSWF's as follows ;

a SSWF $w^S(s\,;\,s_1)$ is such that $S(z\,;\,w^S(s\,;\,s_1))$ which solves

$$\max_{s} U(w,s\,;\,z) = w^S(s\,;\,s_1) - c(z)s$$

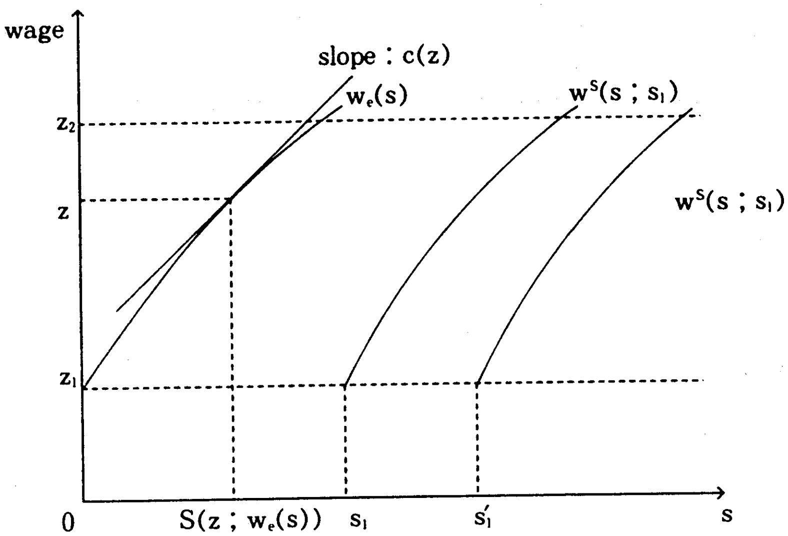
also satisfies

$$w^S(S(z\,;\,w^S(s\,;\,s_1)\,:\,s_1) = z$$

where $w^S(s\,;\,s_1) = z_1$

Here we will not repeat the details of characterizing this set of wage functions, which was done in Spence(1973). Instead we will show them on the graph. A set of wage functions which have the above properties are shown in Figure 2.

Figure 2



Note that $w^s_s > 0$ and $w^s_{ss} < 0$. And we assume for simplicity that $w^s(s ; s_1) = 0$ for any $s \in (0, s_1)$, and that $w^s(s ; s_1) = z_2$ for any $s \in (S(z_2 ; w^s(s ; s_1)), \infty)$, where z_2 is the highest productivity. Note again that thesewage offers for the signals that do not belong to $T(w^s(s ; s_1))$ would not matter as far as $T(w^s(s ; s_1)) = [s_1, S(z_2 ; w^s(s ; s_1))]$. Under the assumption of constant returns to scale in signalling technology, all the wage functions for the different values of s_1 are prarllel to each other. But they differ from each other in the initial signals s_1 for the lowest type. Since the signalling activity is not productive in this model, we can see that $w^s(s ; 0)$ is the most efficient one in the sense that each type of a worker chooses the minimum signal that reveals his productivity. For notational simplicity, let us deonte the most efficient SSWF $w^s(s ; 0)$ and $G(w^s(s ; 0))$ by $w_E(s)$ and G_E, respectively, Then

$$w_E(s) = z(s ; G_E) \text{ for all } s \tag{8}$$

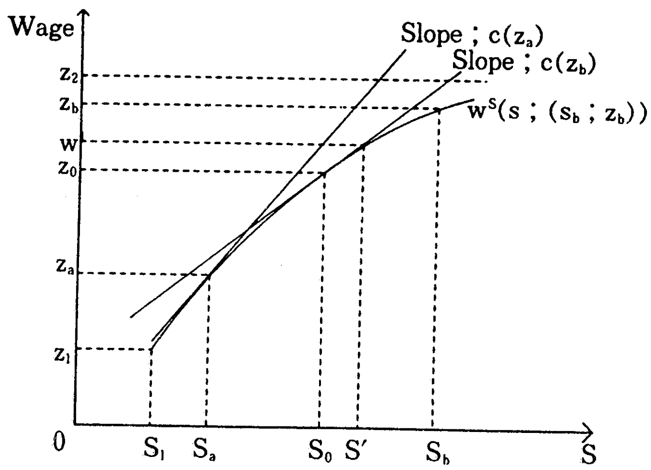
because $w_E(s) = z_2$ for all $s \in (S(z_2 w_E(s)), \infty)$. The straight line in Figure 2 is an indifference curve for a z-type worker. And its slope represents the unit cost of signal for the worker, c(z). So the indifference curve for a z-type worker will be flatter as z gets larger. Given $w_E(s)$, a z-type worker wll choose his optimal signal $S(z\,;\,w_E(s))$ and reveal his productivity as is shown in Figure 2. Also let us denote the wage function $w_E(s)$ restricted to the $[0, S(z_2\,;\,w_E))]$ by $w_E^r(s)$.

Similarily, we can characterize any PSWF $w^P(s)$. Figure 3 shows an example of PSWF $w^P(s)$:

$$
\begin{aligned}
w^P(s) &= w^S(s\,;\,s_l) && \text{for } s_l \leq s < s_a \\
&= z_a && \text{for } s_a \leq s < s_0 \\
&= z_0 && \text{for } s_0 \leq s < s_b \\
&= w^S(s\,;\,(s_b, z_b)) && \text{for } s_b \leq s \leq s_2
\end{aligned}
$$

where $z' = \int_{z_0}^{z_b} z dF(z)/[F(z_b) - F(z_a)]$ and $w^S(s\,;\,(s_b, z_b))$ is a SSWF which starts to offer z_b for a signal s_b. Given this wage function $w^P(s)$, the workers of productivity between z_a and z_b are going to choose s_0 and get their average productivity z_0.

Figure 3



So far we have characterized a set of Spencian equilibria in the context of the extended strategy space. Next we will refine these equilibria by using the characteristics associated with the extended strategy space. The key question in the refinement of Spencian equilibria is how firms would react to an out-of -equilibrium signal or an out-of -equilibrium distribution of signals. Here we will maintain the assumption that each worker always chooses his optimal signal under any circumstance. Then the only reason why firms could see an out-of-equilibrium signal or an out-of-equilibrium distribution of signals is that workers could have an out-of equilibrium conjecture(or the corresponding expected wage function). This seems to be very reasonable if we consider the uncertainty that workers have about the future wage offers for different signals.

Thus an out-of equilibrium distribution of signals or an out-of-equilibrium signal can be rationalized if we can figure out a conjecture of workers which would generate the signal in that distribution of signals. Then the question is whether any signal in any distribution of signals can be rationalized by the optimal behavior of a certain type of a worker based on a common conjecture. We can establish the following proposition about that.

Proposition 2

Any signal in any distribution of signals can be rationalized by the optimal behavior of a certain type of worker given a conjecture or given the following expected wage function $w^e(s)$; for any $m \geq 0$,

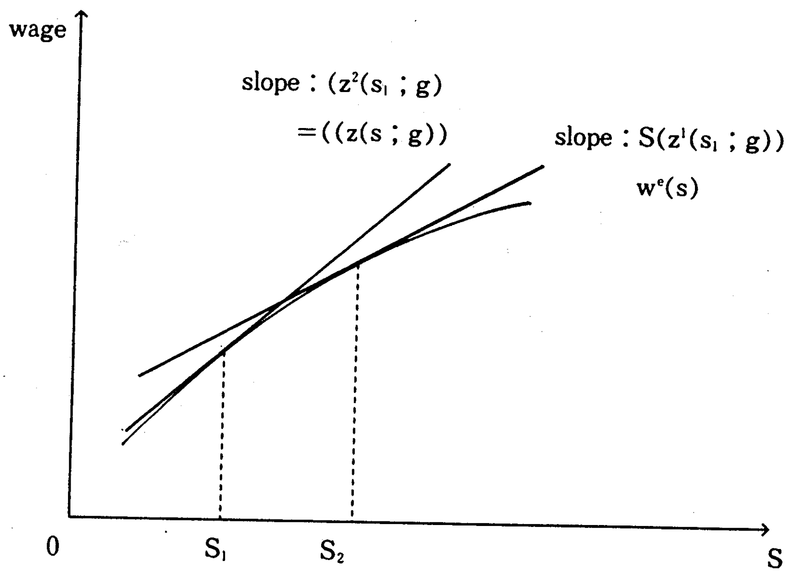
$$w(s) = m + \int_0^s c(z(s' ; g))ds'$$

⟨proof⟩

Consider any distribution g(s) of signals. Then we can determine z(s ; g) by the Matching Process (5) accordingly. Then the proof will be compelete if we show that any signal could be the optimal signal $S(z ; w^e(s))$ of a certain type of a worker given $w^e(s)$, and that the optimal signals of all types of workers given $w^e(s)$ confirm $z(s\,;\,g)$ for all s, i.e., $z(s\,;\,g) = S^{-1}(s\,;\,w^e(s))$ for all s. We know that $w^{e'}(s) = c(z(s\,;\,g))$ and $w^{e''}(s) = c'\ z_s \leq 0$ if $w^e(s)$ is differentiable in s (or if $z(s\,;\,g)$ is differentiable in s). Since $z(s\,;\,g)$ is positive and differentiable almost everywhere, $w^e(s)$ is increasing, continuous and concave in s. If a Z_a-type worker chooses his optimal signal $S(z_a\,;\,w^e(s))$ given $w^e(s)$ as follows

$$\max_{s}\ w^e(s) - c(z_a)s,$$

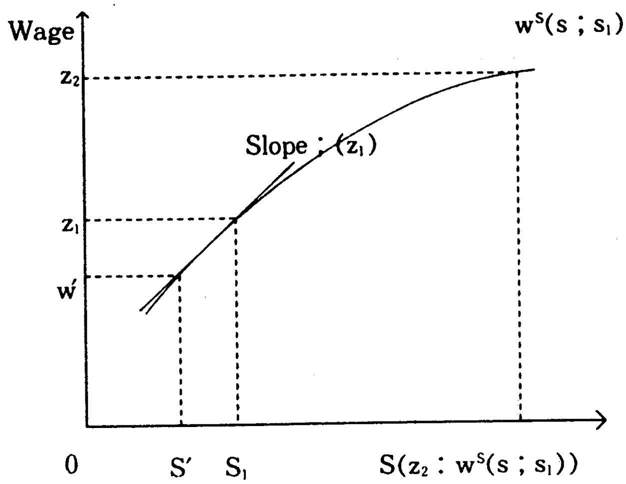
Figure 4



$S(z_a\,;\,w^e(s))(=s_a)$ will be determined such that $w^{e'}(s_a) = c(z(s_a\,;\,g)) = c(z_a)$ if $w^e(s)$ is differentiable at s_a. So $z(s\,;\,g) = S^{-1}(s\,;\,w^e(s))$ if $w^e(s)$ is differentiable at the signal s. If g(s) is constant between s_1 and $s_2 (s_1 < s_2)$so that the probabilty measure of any signal in $[s_1, s_0]$ will be a straight line with the slope $c(z(s_1\,;\,g))$ as in Figure 4.

Since the $z(s_1\,;\,g)$-type workers, whose portion of the whole population is negligible, are indifferent between s_1 and s_0, any signal in $[s_1, s_0]$ can be explained as the optimal signal of the $z(s_1\,;\,g)$-type worker. Suppose $w^e(s)$ is not differentiable at so as in Figure 5. Since $w^e(s)$ is continuous and concave, the workers between $z^1(s_0 ; g)$ and $z^2(s_0 ; g)$(defined in (5)) will choose s_0. So $z(s ; g) = S^{-1}(s ; w^e(s))$ if $w^e(s)$ is not differentiable at the signal s. Thus $z(s ; g) = S^{-5}(s ; w^e(s))$ for all s in g(s). Q.E.D.

Figure 5



The Propositions 1 and 2 imply that firms can always associate the expected productivity $z(s ; g)$ with any signal in any distribution of signals(by the Matching Process (5)), assuming that each type of a worker chooses his optimal signal given a certain conjecture(or given a certain expected wage function). Given this fact, we require that in equilibrium each firm should offer a contract $W^O(s ; g)$ which is optimal with respect to any signal in any distribution of signals(or with respect to any corresponding productivity-signal relationship). Then we can see that $W^O(s ; g)$ is subgame rationalizable in the sense of Pearce(1984) and Bernheim(1984). By the competition among firms, therefore, the optimal contract $W^O(s ; g)$ will be such that

$$W^O(s ; g) = z(s ; g) \qquad \text{for all s and g} \qquad (9)$$

Note that (9) is just the definition of the rank-order contract $W^R(s ; g)$. Thus the rank-order contract is the unique optimal contract that can support an equilibrium in this model. Here it should be noted that the existence of the optimal contract $W^0(s\,;\,g)$ is due to the expected strategy space of firms, which enables firms to adjust their strategies to any out-of-equilibrium situation according to the information revealed by the actual signal distribution. Since the conjecture of workers should be confirmed in equilibrium, an equilibrium can be defined as follows :

An equilibrium $w^*(s)$ is a realized self-fulfilling wage function of the optimal conrtact $W^0(s\,;\,g)$

Notice that this equilibrium is subgame rationalizable in the sense of Pearce and Bernheim, which can be compared with the subgame perfection proposed by Selten (1975). Selten relies on the possible mistakes players can make given their fixed conjecture, while this model relies on the uncertainty that the conjecture of workers involves.

By (9) the equilibrium $w^*(s)$ is going to be a realized self-fulfilling wage function of the rank-order contract $W^R(s\,;\,g)$. Thus we can characterize the equilibrium as follows.

Proposition 3

Ther exists a unique equilibrium in model Ⅰ, which is the most efficient(realized) SSWF $w_E^f(s)$.

⟨proof⟩

From (8) and the definition (6) of the rank-order contract $W^R(s\,;\,g)$, we can see that

$$\begin{aligned} w_E(s) &= z(s\,;\,G_E) \qquad \text{for all } s \\ &= W^R(s\,;\,G_E) \end{aligned}$$

That is, $w_E(s)$ is the self-fulfilling wage function of $W^R(s\,;\,g)$. Therefore $w_E^R(s)$ is an equilibrium. Q.E.D.

It should be noted that the competition among workers in choosing signals under the rank-order contract does not allow and pooling among different types of workers to be viable. Whenever a set of different types of workers chooses the same signal s_0 and are paid their average productivity, $W^R(s ; g)$ will pay the highest productivity of the pooling group for any signal higher than s_0. Thus the marginal benefit for a worker of increasing signal at s_0 will be infinite, which makes the pooling unstable. So the competition among workers in choosing signals under $W^R(s ; g)$ plays an important role in determining the nature of an equilibrium.

So far we showed that there exists a unique equilibrium in model Ⅰ, Now let us turn to model Ⅱ, which relies on the competition among firms in offering the contracts to resolve the multiplicity of equilibria in Spencian model.

Ⅳ. The Model Ⅱ

In model Ⅱ, each firm first announces its wage contract that specifies how its wage offers to workers will be based on their signals before they choose signals. And after workers choose signals each firm implements its conrtact as announced. It has been shown in R-R-S that there may not exist a competitive Nash equilibrium in this type of model. However, we argue that there always exists a unique equilibrium if the strategy space for a firm is extended to distributions of signals, and that the equilibrium is supported by the rank-order contract $W^R(s ; g)$.

First of all, we would like to justify the extended space of a firm in model Ⅱ. It is applicants and implement their announcements. So it is after a distribution of signals is realized and observed by workers and firms are going to be implemented. Knowing this fact, a firm will want to announce its wage contract based on distributions of signals as well as on individual signals as described in (2).

Without any restriction on the set of possible wage contracts firms can announce however, a wage contract in this extended strategy space may be too complicated for a firm to announce. Here we impose a following reasonable restriction C on the set of wage contracts firms can announce :

C : For any given s, $W^i(s\,;\,g)$ is continuous on M, where(M, d_M) is a metric space with a metric d_M defined in (3). The restriction C implies that an wage offer for a certain signal should change continuously as the signal distribution varies. So the competition among firms is restricted to the set of contracts satisfying C.

The crucial aspect of the moedl Ⅱ is the externality that each contract exerts on the profits of the other contracts through its possible impact on the productivity-signal relationship. This is the basic reason for the possible nonexistence of an equilibrium in R-R-S. In this model, the productivity-signal relationship will also change to the extent that a deviant contract affects the market wage contract. However, the effect of the change in productivity-signal relationship(or the change in distributions of signals) on the profits of the other contracts is not so straightforward as in R-R-S. This is because the wage offers of the other firms for each signal can change as the underlying signal distribution change due to the introduction of the deviant contract. If the underlying signal distribution changes very little, however, the wage offers of the other firms will also change very little because of the continuity restriction C. Since we can show by the folliwing Lemma that the signal distribution changes continuously as the market wage contract changes, there is still a room for a deviant firm to take advantage of the other contracts(as in R-R-S) if the diviant contract changes the market contract only slightly. This is the intuitive reason why the competition among firms with respect to contracts in the extended strategy space is also effective in reducing the set of Spencian equilibria as in R-R-S.

Lemma 1

$G(W(s\,;\,g^e)$ is continuous in $W(s_l\,;\,g^e)$ for any signal s_l.

The proof of Lemma 1 is relegated to Appendix. But the intuition is the following. Since c(z) is continuous in z_l, the set of types of workers choosing s_l will be continuous in $W(s_l\,;\,g^e)$. Also since F(z) is continuous, the portion of workers choosing s_l will be continuous in $W(s_l\,;\,g^e)$ so that $G(W(s\,;\,g^e)$ is continuous in $W(s_l\,;\,g)$.

Given the optimal behavior on the part of workers and the wage contrats announced by the other firms, a firm tries to announce its wage contract that can profitably attract workers. And the profitability of a certain contract will depend on the pattern of an optimal response of workers(or the distribution of signals), which will also depend on their common conjecture g^e. Then we can define an undominated contract $W^u(s ; g)$ as follows :

$W^u(s ; g)$ is an undominated contract if

(i) there does not exist a contract $W'(s : g)$ such that

$$\pi(W', W^u ; g^e) \geq \pi(W^u, W^u ; g^e) \text{ for all } g^e,$$

$$\pi(W', W^u ; g^e) > \pi(W^u, W^u ; g^e) \text{ for some } g^e, \quad (10a)$$

(ii) an entrant firm cannounce a contract $W'(s ; g)$ such that

$$\pi(W', W^u ; g^e) \geq 0 \text{ for all } g^e$$

$$\pi(W', W^u ; g^e) > 0 \text{ for some } g^e \quad (10b)$$

where $\pi(W^1, W^u ; g^e)$ is the profit of a contract W' given an upper envelope W^u of all the other contracts and a common conjecture g^e. Note that the condition (10b) excludes the possibility of a profitable entry.

In the models where firms move first by offering their contracts before workers choose signals, it is normally assuned that workers are going to respond optimallly to any set of contracts(including out-of-equilibrium contract)offered(R-R-S, Stiglitz-Weiss(1983)). In this model, workers are also supposed to respond optimally to any market contract too. The basic problem is, however, that as the market contract changes by the introduction of a deviant contract W', the common conjecture of workers may change accordingly. Then the profitability of the deviant contract W', which will be determined by the pattern of the optimal response of workers to the new market contract, will depend on the new common conjecture.[3] The conditions (10) are based on the plausible rule that a deviant contract will be introduced if it can make at least the same profits(10a) or zero profits(10b) under any some conjectures. The requirements(10) for a deviant contract to be introduced in the market might be considered as too strong(or the restriction on the set of undominated contracts is too weak as a result), because a deviant could introduce a contract $W''(s ; g)$ if $\pi(W''$,

W^u ; g^e) > 0 for some g^e although $\pi(W'', W^u ; g_1{}^e) < 0$ for some other $g_1{}^e$. As we still see later, however, any weaker requirements than (10)(or the stronger restriction on the set of undominated contracts) will not affect the result of this section-the unique existence of an equilibrium.

Let D(W) be a set of possible signal distributions generated by the optimal response $G(W(s ; g^e))(=G(W, g^e))$ of workers to a market contract W(s ; g) for all possible conjecture g^e. Then the condition (10) of an undominated contract imply the following :

Proposition 4

Suppose W(s ; g) is an undominated contract. Then for any actual signal distribution $G(W,g^e) \in D(W)$,

$$W(s ; G(W,g^e) = z(s ; G(W,g^e)) \text{ for all } s \in T(W(s ; g)) \qquad (11)$$

⟨proof⟩

Suppose not. If there exists a signal $s_1 \in T(W(s ; g_1^e))$ for some belief g_1^e such that $W(s_1,g_1) < z(s_1,g_1)$, where $g_1 = G(W,g_1^3)$. Then we can consider a contract W' (s ; g) such that W' (s ; g) = 0 for all $(s ; g) \neq (s_1 ; g_1)$ and $W'(s_1 ; g_1) = W(s_1 ; g_1)$. Since this deviant contract will not change the market contract W(s ; g), $\pi(W', W ; g_1^e) > 0$ and $\pi(W', W ; g^e) = 0$ for $g^e \neq g_1^e$ This implies that W(s ; g) is not an undominated contract. On the other hand, suppose there exists a signal $s_1 \in T(W(s ; g_1^e))$ for some belief g_1^e such that $W(s_1,g_1) > z(s_1,g_1)$, where $g_1 = G(W,g_1{}^e)$Then we can also consider a contract W' (s ; g) such that $W'(s_1,g_1) = 0$ and W' (s ; g) = W(s ; g) for all $(s ; g) \neq (s_1 ; g_1)$. Since W' (s ; g) does not change the market contract W(s ; g), we can see that $\pi(W_1,W ; g_1^e) > \pi(W_1,W ; g_1^e)$ and $\pi(W', W ; g^e) = \pi(W_1,W ; g^e)$ for all $g^e \neq g_1^e$, which contracts the condition (10b).

Q.E.D.

The Proposition 4 implies that any undominated contract W^u should have zero profit for any actual signal distribution $G(W^u,g^e) \in D(W^u)$. The converse is not true, of course.

Now let us consider the rank-order contract $W^R(s ; g)$. First we can show by the following lemma that $W^R(s ; g)$ is continuous in g.

Lemma 2

$z(s\,;\,g)$ is continuous in g for any s.

The proof of this lemma is relegated to the Appendix.

Next we can establish the following :

Proposition 5

The rank-order contract $W^R(s\,;\,g)$ is an undominated contract.

⟨Proof⟩

From Proposition 1 we can see that the rank-order contract $W^R(s\,;\,g)$ always pays a worker his expected productivity. So $W^R(s\,;\,g)$ always gives zero profit for any type of optimal response by workers, and thus there will be no other wage contract that can profitably attract workers given $W^R(s\,;\,g)$ for any conjecture g^e. Q.E.D.

Riley(1979) showed that when firms move first by offering contracts before workers choose their signals, there does not exist any Nash equilibrium contract. The basic reason for this non-existence lies in the externality which is inherent in his model, as we mentioned before. In this model, however, each firm could announce the rank-order contract $W^R(s\,;\,g)$ and protect its profitability from any possible externality that may be caused by the other contracts. Thus there is no problem of non-existence of a Nash equilibrium contract here. This is also true even if we impose weaker requirements than (10) for the deviant contract to be introduced(or stronger restrictions on the set of undominated contracts) This is because the rank-order contract $W^R(s\,;\,g)$ is always consistent with the information (5) that a signal distribution generates through the induced competition among workers in their choices of signals, so that there could possibly be no contract that can increase its profits or make positive profits for any g^e given $W^R(s\,;\,g)$.

However there could be other undominated contracts. If $W(s\,;\,g)$ satisfies

$$W(s\,;\,g) = z(s\,:\,g) \quad \text{for all } g \in D(W) \text{ and}$$
$$W(s\,:\,g) > z(s\,:\,g) \quad \text{for all } g \not\ni D(W)$$

W(s ; g) will be an undominated contract. In other words, to be an undominated contract, W(s ; g) does not have to satisfy the zero profit condition for all s and g. Thus the undominated contract is not unique in generl in model Ⅱ.

So when we extend the strategy space of a firm to signal distributions, the competition among firms with respect to contracts leads to multiple undominated contracts, rather than its non-existence as in R-R-S. However, this does not necessarily imply the multiplicity of equilibria, because the equilibrium should also be concerned with the competiton among workers in choosing their signals given a market contract announced. In particular, we define an equilibrium in this model as follows.

An equilibrium $w^*(s)$ is a realized self-fulfilling wage function of an undominated contract $W^u(s ; g)$.

Note that the equilibrium is defined not as a market contract itself, but as its self-fulfilling wage function which confirms the conjecture of workers. In R-R-S where firms competitively offer their contracts that are based on individual signals only, workers will always be satisfied with their choices of signals ex-post given a set of contracts offered by firms. This is because when each worker chooses his signal. he does not have to care about what the other workers are going to choose. So the Nash equilibrium in their models can be characterized by a set of contracts is not trivial. Thus the Nash equilibrium should explicitly take into account the best response of each worker given the actions of all the other workers and a set of contracts offered. And it is only when workers expect a self-fulfilling wage function of a certain contract to prevail that each of them is satisfied with his choice of signal ex-post. That is why an equilibrium of this model is defined as a (realized)self-fulfilling wage function of an undominated contract.

Note that every undominated contract does not have its self-fulfilling wage function. In this sense, the competition among workers in choosing their signals, in addition to the competition among firms in announcing their contracts, will further restrict the scope of equilibria.

To characterize an equilibrium of model Ⅱ, we might have to check each undominated contract and see what its self-fulfilling wage function looks like if any.

Since this is almost impossible, however, we will try a different approach by suing Proposition 4. Since an equilibrium is a(realized)self-fulfilling wage function of an undominated contract and every undominated contract should satisfy zero profit condition by Proposition 4, an equilibrium will belong to the set of Spencian wage functions w(s). So we will consider a contract which contains any of the Spencian wage functions as its self-fulfilling wage function, and see if the contract is an undominated one.

Since we know by Proposition 3 that $w^E(s)$ is supported as the self-fulfilling wage function of the rank-order contract $W^R(s ; g)$ which is an undominated one by Proposition 5, the realized wage function $w_E^r(s)$ is an equilibrium in the model Ⅱ. Now we will show that $w_E^r(s)$ is the unique equilibrium by demonstrating that any contract whose self-fulfilling wage function is not $w_E(s)$ is not an undominated one. First we can prove the following :

Proposition 6

Any inefficient SSWF $w_E(s ; s_1)$ (where $s_1>0$) is not an equilibrium.

⟨Proof⟩

Consider an inefficient SSWF $w^s(s ; s_1)$ as shown in figure 5. Then it should be that

$$w^S(s' ; s_1) < w' < z(s' ; G(W^S(s ; s_1)) \text{ and}$$
$$w^B(s'' ; s') \leq z_2$$

where $s' < s_1$, $w' - c(z_1)s' = z_1 - c(z_1)s_1$ and $s'' \geq S(z_2 : w^s(s ; s_1))$. Suppose $W(s ; g)$ is any market contract which has $w^s(s ; s_1)$ as its self-fulfilling wage function :

$$w^S(s ; s_1) = W^*(s ; g^*)$$

where $g^* = G(w^s(s ; s_1))$. Then consider another contract $W'(s ; g)$ such that

$$W'(s ; g) = 0 \text{ for all } g \text{ and } s \neq s'$$
$$W'(s' ; g) = w(s' ; g) + \beta\{z(s' ; g) - W(s' ; g)\} \text{ if } z(s' ; g) \geq W(s' ; g) \geq W(s' ; g) \text{ and } = z(s' ; g) \text{ if } z(s' ; g) < W(s' ; g)$$

where $\beta < 1$. We can see that $W'(s;g)$ is continuous in g for any s because $W(s;g)$ and $z(s;g)$ are continuous in g for any s. Also we can see that $W'(s;g)$ will never make losses for any belief g^e. So we only have to show that $\pi(W',W;g^e)>0$ for some g^e Suppose $W''(s;g)$ is a new market contract formed by the introduction of the deviant contract $W'(s;g)$. And let β be such that

$$W''(s';g^*)+\beta\{z(s;g^*)-W''(s';g^*)\}=w'$$

Then the optimal response $G(W'',g^*)$ with respect to the belief g^* will depend on β So let us denote $G(W'',g^*)$ by $H(\beta)$. Then $H(\beta)=g^*$ if $\beta \leq \beta$, and $H(\beta) \neq g^*$ if $\beta > \beta$. From the continuity of the optimal response $H(\beta)$(by Lemma 2), we can see that there exists a $\beta(>\beta)$ which is very close to β, such that $H(\beta)-g^*<\varepsilon$ for any small ε. Then by the continuity of W(or W') and z(sig), there will exist a very small ε or the corresponding β such that

$$z(s';H(\beta))>W(s';H(\beta))$$

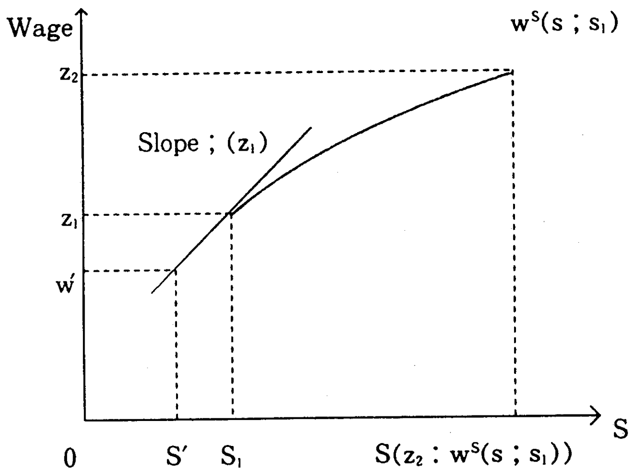
Therefore

$$z(s';H(\beta))>W(s';H(\beta)) \text{ by the definition of } W'.$$

Since $W'(s';H(\beta))>W(s';H(\beta))$, we can see that the deviant contract W' will attract some workers choosing s' given their belief g^*. So $\pi(W',W;g^*)>0$. Q.E.D.

So Porposition 6 excludes all the SSWF's except $w_E(s)$ from the set of equilibria. The intuition behind this proposition is basically the same as that in R-R-S, it is the competition of firms for wage-signal pairs that eliminates all the Pareto-dominated contracts. This competition is effective in their models because each firm takes as given all the other wage-signal offers made by the other firms. In this model each firm also takes all the other contracts as given. But since the strategy space of a firm is extended to signal distributions, a deviant contract will affect the set of all the other wage-signal offers through its possible effect on the actual signal distribution. Suppose a deviant firm offers its contract which affects the market wage contract, and suppose the new conjecture g^e of workers is the self-fulfilling one g^* for the previous market contract as in the above proof. Also suppose the expected wage-signal offers of the deviant given the conjecture g^* can profitably attract workers if g^* were actually realized. If the deviant contract is almost the same as the previous market contract, the actual signal distribution g generated by the introduction of the deviant contract under the conjecture g^* will be very close to g^*. Then by the continuity C of contracts, the actual set of wage-signal offers by the other firms, which is based on the actual signal distribution g, is almost the same as the expected set of their wage-signal offers based on the g^*. Then the deviant contract could still be profitable given

Figure 5



the actual signal distribution g. In other words, since the actual signal distribution g under the previous self-fulfilling belief g^* could be very close to g^*, the competition of firms for signal-wage pairs(as in R-R-s) could take place in the neighborhood of the self-fulfilling wage function of the previous market contract. The important factor that makes possible this competition of firms for wage-signal pairs is the continuity C of contracts. And that competition of firms eliminates any inefficient SSWF as an equilibrium in the same way as R-R-S did. By the same reasoning, we could also exclude any PSWF from the set of equilibria.

Proposition 7

Any PSWF $w^P(s)$ is not an equilibrium.

⟨proof⟩

Consider a PSWF $w^P(s)$ which pools different types of workers(between z_a and z_b) who choose the same signal s_0 (Figure 3). Then it should be the case, as can be seen in Figure 3, that

$$w^P(s') < w'$$

where $s' \in (s_0, s_b)$ and $w' - c(z_b)s' = z_0 - c(z_b)s_0 (= z_b - c(z_b)s_b)$.

Suppose $W(s\,;\,g)$ is any contract which has $w^P(s)$ as its self-fulfilling wage function :

$$w^P(s) = W^*(s\,;\,g^*)$$

where $g^* = G(w^P(s))$. Then consider a contract $W'(s\,;\,g)$ such that

$$W'(s\,;\,g) = 0 \text{ for all } g \text{ and } s \neq s'$$

$$W'(s'\,;\,g) = W(s'\,;\,g) + \beta\{z(s'\,;\,g) - W(s'\,;\,g)\} \text{ if } z(s'\,;\,g) \geq W(s'\,;\,g) \text{ and}$$
$$= z(s'\,;\,'g) \text{ if } z(s'\,;\,g) < W(s'\,;\,g)$$

where $\beta < 1$. Then we can see that $W'(s\,;\,g)$ is continuous in g for any s, because $W(s\,;\,g)$ and $z(s\,;\,g)$ are continuous in g for any s. Also $\pi(W', W\,;\,g^e) \geq 0$ for any g^e.So we only have to show that $\pi(W', W\,;\,g^e) > 0$ for any g^e. Then, by following the same procedure used in the proof of Proposition 6, we can show that $\pi(W', W\,;\,g^*) > 0$.

The Proposition 7 shows that as in Riley, we can also eliminate any PSWF $w^P(s)$ by the competition among firms for contracts. Since all the inefficient SSWF's and all the PSWF's are removed from the set of equilibria, $w_E^r(s)$ turns out to be the only equilibrium in this model by the propositions 3 and 5. That is, Proposition 8

There exists a unique equilibrium $w_E^{I}(s)$ in model Ⅱ, in which each type of a worker is completely separated from the others in the most efficient way.

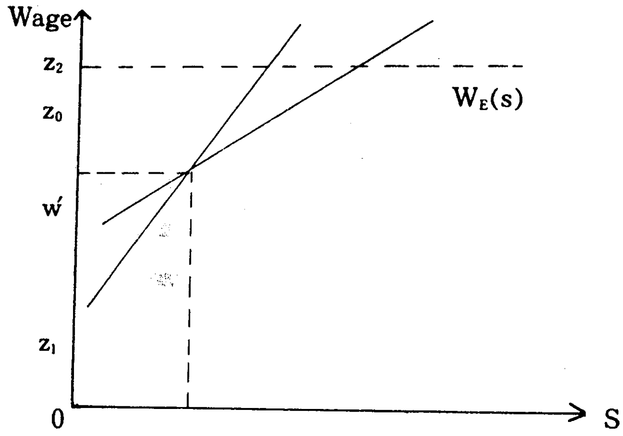
Then a natural question that could be raised is why the competition of firms for wage-signal pairs is not effective in this case while the separating contract could not survive that competition in R-R-S. The basic reason for a profitable deviant contract because it is always consistent with the information (5) a signal distribution generates through the induced competition of workers.

Ⅴ. Concluding Remarks

In this paper, we considered the two types of competitive signalling models whers firms condition their wage offers on distributions of signals as well as on the absolute levels of signals. This extension of the strategy space of firms has some important implications in each of the two models, which lead to the unique equilibrium where each type of a worker is completely separated from the others in the most efficient way. The first implication is that firms can adopt the rank-order contract which is always consistent with the information(about the productivity-signal relationship) revealed by a distribution of signals. Then the rank-order contract will be optimal with respect to any signal distribution in model Ⅰ, and its profit will not be affected by the other contracts(i.e., externality-free) in model Ⅱ. This fact enables us to reduce the Nash equilibria to the unique robust equilibrium in model Ⅰ, and also guarantees the existence of an equilibrium in model Ⅱ. The second implication of the extended strategy space is that a market wage contract in this extended strategy space induces the competition among workers in their choices of signals, which determines the nature of equilibrium. In particular, we showed that the competition among workers under the rank-order contract does not allow any pooling among different types of workers to be viable, and that it leads to the complete separation among different types of workers in the most efficient way. The last implication, which is relevant in model II, is that the competition among firms with respect to contracts in the extended strategy space eliminates many Pareto-dominating contracts as in R-R-S.

Finally, let us consider the optimaltity of the equilibrium briefly. Although the equilibrium $w_E^I(s)$ is the most efficient separating one, it could be Pareto-dominated by a pooling contract in a way that some workers can be made better off without the other workers being worse off. That is exactly the case where there does not exist any Nash equilibrium in R-R-S as we have discussed before. We have illustrated this case in Figure 6.

Figure 6



Here whether z_0 is greater than w' or not is critical for the optimality of the equilibrium in my models or for the existence of an equilibrium in Riley's model. This depends on the cost of signllling for each type and the distribution of types of workers. This is because these two factors determine the welfare costs the more productive workers should bear for their separations. If the cost of signalling is very high and or the population of the more productive workers is large relative to that of the less productive workers, the more productive should bear relatively large welfare costs to separate themselves from the less productive. Then a separating contract based on absolute levels of signals only is more likely to be Pareto-dominated by a pooling contract. In particular, Riley(1979) showed that there always exists such an unravelling pooling contract when there is a continum of types of workers. This implies that the equilibrium $w_E^i(s)$ in each of the two models in not pareto optimal. And we should that the two-sided competition both on the part of firms and on the part of workers keep a pooling contract from being offered, although it could lead to Pareto improvement.

Footnotes

1) Precisely speaking, we need $W(s\,;\,g^e)$ to be continuous in s for the existence of $s(z\,;\,W(s\,;\,g^e))$. And for a market wage function $W(s\,;\,g^e)$ to be continuous, each firm should be restricted to announce only a continuous wage function for any signal distribution. Instead of giving a restriction on the set of wage functions a firm can announce, we will pretended that the minimum or maximum of an open interval of signals exists. For example, when a worker would like to choose the smallest one of all the signals belonging to an open interval(s^1,s^2), he is assumed to choose $s^*=\min\{s^1 \mid s^2\in(s^1,s^2)\}$. The s^* could be interpreted as a signal which is so close to s^1 that every worker takes the signals between s^1 and s^* as the same.

2) We could assume that all firms share a common belief about the beliefs workers are going to have given any possible market wage contract. Suppose W^* is a market contract which is an upper envelope of a deviant contract W' and the set W of the existing contracts. And suppose firms share the belief $G(W^*)$ which indicates the set of possible beliefs workers are going to have given W^*. Let $E\pi(W',W\,;\,g^e)$be the expected profit of a deviant contract W' given the belief $g^e\in G(W^*)$. Then the main proposition of this model-the unique existence of anequilibrium-will not be affected if we suppose that a deviant would offer its contract W' when $\pi(W',W,g^e)>0$ for some g^e and $\pi(W',W\,;\,g^e)\geq 0$ for all g^e even if $E\pi(W',W\,;\,g^e)=0$.

3) For simplicity, here we set zero wage offers for signals that are not chosen by workers given $w(s)$. Actually any wage offer could be set for $s\in T(w(s))$ as far as (8) is satisfied.

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