A Study on Environment Quality in a Marxian Model of Reproduction

Kyung-Seop Shim *

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Ⅰ. Introduction

This paper reports on research which aims to introduce waste emissions and environmental pollution into Marxian theory of how capitalist society reproduces and expands itself. That is, my ultimate goal is to see how the contemporary issue of waste disposal and environmental quality affect the specification and implications of the political economy of capitalism originally formulated by Karl Marx more than a century ago.[1]

The theoretical framework adopted here has several features worth noting. First, each sector of production has a single technique available to it at any moment in history.[2] Second, there is a

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* Professor of Economics, DanKook University in Seoul

1) For an introduction to his theory of reproduction and capital accumulation, see Karl Marx(1967, Vol. Ⅰ, Part Ⅶ)

2) As Rosenberg has pointed out, a limited number of techniques(perhaps only one) exists at any moment in any particular sector and new methods requiring substantially different input proportions are

discrete time period of capital turnover common to all sectors which must elapse between the purchase of productive inputs and the sale of finished products. In explicit recognition of the physical law of mass-energy conservation, we introduce a conception of technology which emphasizes the propensity of a technique to produce toxic, persistent wastes as well as marketable commodities.3)

This analysis assumes that there is an infinitely elastic supply of the work force. The venerable notion of a subsistence wage should not be interpreted as a physiologically-determined minimum which leaves workers on the brink of starvation. Rather, its level is determined by the physical quantities of various wage goods which workers require in order to reproduce their manual and mental capacities to labor, given the present division of labor and social organization of work and consumption.4)

A final distinctive feature of this analysis is that it utilizes two different accounting systems. At one level, linkages among physical variables such as the environmental concentration of pollutants, the utilization rates of labor and means of production, and the output rates of various commodities are taken into account.

This set of variables is generally familiar to economists but the second is less so, namely a set of value concepts such as surplus value, variable capital and constant capital, all of which entail measurement of the hours of labor directly and indirectly embodies in commodities during their production.

Ⅱ. Specification of Model

Let us consider the physical relationships of the model. Production takes place in two different sector, the first producing means of production(e.g., tools, raw materials)and the second producing

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discovered via an historical process of technical innovation. See Nathan Rosenberg(1976, ch. 4)

3) See R. Ayres and A. Kneese(1969) on the joint production of wastes and commodities.

4) As Marx himself put it, "[The worker's] natural wants, such as food, clothing, fuel, and housing, vary according to the climatic and other physical conditions of his country. On the other hand....there[also] enters into the determination of the value of labor-power a historical and moral element." Karl Marx(1967, Vol. Ⅰ, p.171).

means of consumption. Each sector has a single method of production which can be described by the vector.

$$(a_i,\ l_i,\ r_i),\quad i=1,\ 2 \tag{1}$$

where a_i = the amount of some physically homogeneous means of production require in the i^{th} sector per unit of its gross output.

l_i = the number of hours of work directly required in the i^{th} sector per unit of its gross output, and

r_i = the physical amount of a homogeneous pollutant emitted per unit of gross production in the i^{th} sector, and where both sectors are assumed to have periods of capital turnover of the same duration, call it one "day" Let

δ = the fraction of the stock of pollutants already in the environment at the end of an earlier period of production which decays to non-toxic forms during the following production period.

Note that physical means of production are assumed to consist strictly of materials used up during the period of production and no durable capital goods are required to produce either product.[5] By assumption, the a_i, l_i, and r_i technical parameters are positive, whereas δ is positive and less than one.

Turning next to the supply of labor, we specify that

$$N < N^* \tag{2}$$

where N = the total number of workers employed during a period and

N^* = the number of workers available for employment[6]

5) the inclusion of durable capital although certainly desirable, would complicate the analysis greatly by rasing the issue of capital goods of different vingages. Whether the simplifying assumption of circulating capital gravely flaws this model remains to be seen.

6) This assumption of a labor-surplus economy would hold in those developing capitalist nations where large numbers of wage workers can be recruited from pre-capitalist sectors of the economy or during much of the business cycle in develped capitalist countries.

The relation between the total number of hours of employment per day, call it L, and the number of workers employed is given by

$$L = T \cdot N \qquad (3)$$

where T = the customary length of the workday, i.e., standard number of hours worked daily per worker

The total number of hours of employment per day is dicatated by the sectoral gross output rates and by the labor-output ratios corresponding of the present state of technology, i.e.,

$$L = l_1 x_1 + l_2 x_2 \qquad (4)$$

where x_2 = the gross output rate in the i^{th} sector.

If production is to take place during the current period, then society must have inherited from the previous period adequate supplies of produced means of production and of the wage goods required to support employed workers at the subsistence level :

$$x^0_1 \geq a_1 x'_1 + a_2 x'_2 \qquad (5)$$

and

$$x^0_2 \geq bN' = \frac{b}{T}(l_1 x'_1 + l_2 x'_2) \qquad (6)$$

where b = the number of wage goods required daily by a worker to reproduce his or her capacity to work, and where prime and zero superscipts specify a dynamic link between the present period the previous period, respectively[7].

Taking the production of waste residuals into account,

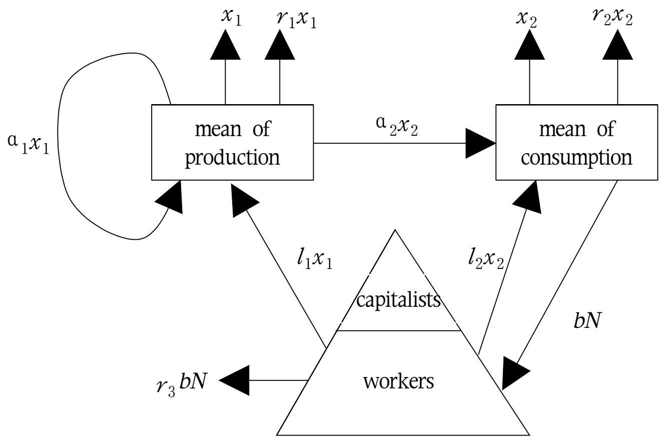
7) One might ask what happens to those workers who are not employed at the subsistence aily wage. The answer depends on the particular capitalist country and its stage of socio-economic development, e.g., return to peasant agriculture, self-employment as a pretty trader or craftperson, begging, thievery, starvation, unemployment compensation, the poor house, etc.

$$R = r_1 x_1 + r_2 x_2 + r_3 bN, \quad (7)$$

where R = the total output of waste matter and energy during a period, and where r_3 is the amount of the homogeneous pollutant emitted per wage good comsumed within the household sector.[8)]

Finally, the environmental concentration of pollutants, P, is determined in the follwing way:

$$P' = P' + (1-\sigma) \cdot P^{o}. \quad (8)$$



<Figure 1>

This complete set of physical input-output relationships is depicted in Figure 1.[9)]

Now for the value relations which introduce for the reasons already mentioned.

First, the value of a commodity, λ, is defined as

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8) Given the enormous variety of physically distinct forms of matter and energy emitted into the environment, it is not immediately clear how one should calculate this aggregate. One supposes that it would be a weighted average of the emission flows of specific pollutants, each weighted by some physical measure of its unit toxicity. Because of bio-chemical reactions among pollutants, however, these weights might have simultaneously.

9) This figure points out another simplifying assumption of this model, namely that capitalists do not consume produced commodities. Let us suppose that capitalists enjoy caviar-flavoured manna which God gratuitously pops into their mouths in recognitionof their stewardship over the wealth of society. This assumption is empirically plausible if the bulk of property income is accumulated and not personally consumed.

$$\lambda i = \lambda_1 a_1 + l_i, \quad l = 1, 2. \tag{9}$$

That is, the value of a commodity is equal to number of hours of work directly embodied in a unit of that commodity plus the number of hours indirectly necessary to reproduce means of production consumed during its production.

The total value of capital employed by capitalists, K, consists of two components. Variable capital, V, is the value of wage payments to employed workers during the period of production and prior to the appropriation and market sale of the output by the capitalists. Constant capital, C, on the other hand, is the value of the physical means of production productively consumed during that same period of production. That is, in general,

$$V = \lambda_2 bN \tag{10}$$

$$C = \lambda_1 \cdot (\alpha_1 x_1 + \alpha_2 x_2), \tag{11}$$

and

$$K = C + V \tag{12}$$

The value of aggregate output produced during a period is simply

$$Y = \lambda_1 x_1 + \lambda_2 x_2 \tag{13}$$

The workers employed by the owners of capital produce surplus value, S, if the value of aggregate output exceeds the total value of capital advanced during a period

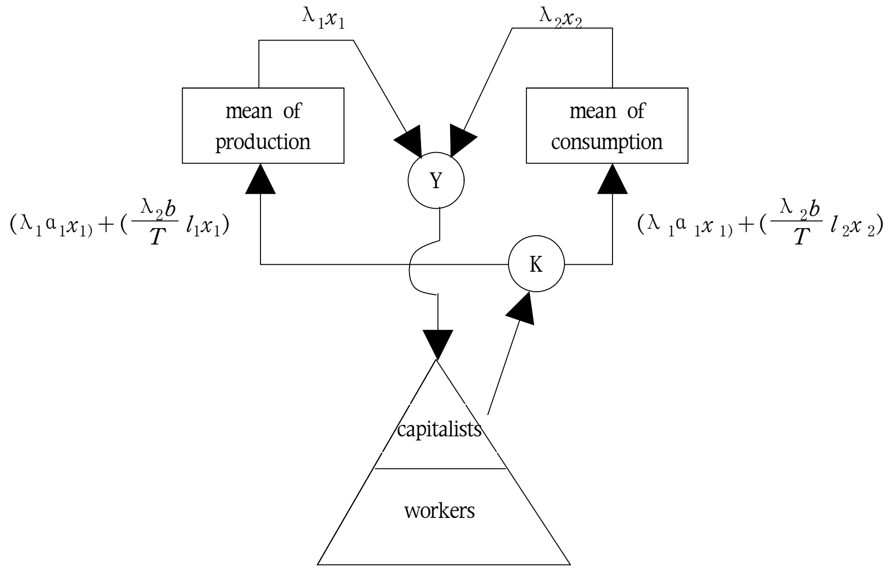
$$S = Y - K. \tag{14}$$

The hypothesis of Marx was that this surplus of hours of labor beyond those required to reproduce employed workers and to replace those means of production used up during production was the origin of all forms of property income (profits, royalties, rents, interest, etc.) if that surplus value could be realized via the market exchange of the finished commodities.

Finally, the value rate of profit during a period is given by

$$\pi = S/(C+V) \tag{15}$$

These value flows are summarized in Figure 2.



<Figure 2> Turnover Aggregate Capital

* This assumes that surplus value produced is fully realized via commodity exchange.

Ⅲ. Existence and Determinants of Equilibrium Output

In this analysis, three sets of conditions have to be satisfied in equilibrium. First, both sectoral output rates must be positive. Second, the physical supplies of both commodities inherited from the previous period must be fully consumed during the present period. Third, the precise total and composition of capital thrown into production during the current period must be just sufficient to guarantee that the outputs of the previous period are sold. That, the complete list of equilibrium conditions reads as follows :

$$x_1 > 0,\ x_2 > 0, \tag{16}$$

$$x_1^0 = \alpha_1 x_1' + \alpha_2 x_2', \tag{17}$$

$$x_2^0 = bN', \tag{18}$$

$$C' = \lambda_1 x_1^0, \tag{19}$$

and

$$V' = \lambda_2 x_2^0, \tag{20}$$

Solving(17) and (18) simultaneously, one finds that the equilibrium output rates of the two sectors can be calculated as

$$x'_1 = \frac{(a_2/a_1) \cdot [(x_1^0/a_2) - (x_2^0/\frac{b}{T} l_2)]}{1-a} \tag{21}$$

$$x'_2 = \frac{[(x_2^0/\frac{b}{T} l_2) - (ax_1^0/a_2)]}{1-a} \tag{22}$$

where $\alpha \equiv (l_1/a_1)/(l_2/a_2)$

These two equilibrium output rates can be positive only if particular magnitudes of the technical parameters and of the real wage rate, *(b/T)*, co-exist. In particular, if sector Ⅰ is relatively labor-intensive, i.e, $a > 1$, then $x_i > 0$, $i = 1, 2$
if and only if

$$(\frac{b}{T} l_1/a_1) > (x_1^0/x_2^0) > (\frac{b}{T} l_2/a_2) \tag{23 i}$$

is satisfied. If, on the other fand, Sector Ⅱis relatively labor-intensive then both sectoral output rates can be positive if and only if

$$(\frac{b}{T} l_2/a_2) > (x_2^0/x_1^0) > (\frac{b}{T} l_1/a_1) \tag{23 ii}$$

is satisfied.

That is, a substantial difference in sectoral input ratios results in an opportunity fully to utilize inherited input endowments of widely varied proportions via an appropriate adjustment in the composition of aggregrate output. The more similar the input ratios of the two sectors, on the other hand, the narrower the range of input endowment proportions consistent with the possibility of fully employing the entire endowment. To be specific, as the magnitude of a approaches unity, (x_2^0/x_1^0) must approach the particular magnitude of *(b/T)* times (l_2/α_2), a condition determined by the real wage parameters and the technical parameters of the consumer goods sector.

In general, however, the aggregate ratio of inherited supplies of means of cosumption to means of production must fall between the sectoral of means of consumption of wage goods to (direct) utilization of means of production. Otherwise, a disproportionality problem prevents equilibrium.

Let us suppose that the technical and real wage parameters during the current period satisfy the requirements for the existence of an equilibrium which fully employs both produced inputs, i.e., (23 i) or (23 ii) is satisfied depending on which is germane. One can then derive the feasible output set for the current period by ploting (5) and (6) and by discarding that segment of each constraint which is not binding because of a shortage of the other produced input. Note that the supplies of both produced inputs are actually fully employed only if the scale and composition of aggregate output places the economy at point F during the current period.

Ⅳ. Feasible Set of Output Rates

On top of the technical and real wage restrictions just discussed, what additional condition must be satisfied if the economy is to operate at this full-utilization equilibrium? The decisions of the capitalists who own the means of production and who hire the laboring capacities of workers are decisive at this point. Capitalists fully realize the value of last period's aggregate production if, and only if, the total value and composition of capital which they advance this period clears both produced input markets. In other words, constant capital must equal the value of last period's output of means of subsistence. Thus, the complete set of necessary and sufficient conditions to guarantee current production at point F is (23 i) or (23 ii) and both (19) and (20).

If these conditions have been met, then the equilibrium value of aggregate output can be calculated by substituting (21) and (22) into (13) :

$$Y' = \frac{l_1 x_1^0}{1-a_1} + \frac{x_2^0}{b/T} \tag{24}$$

Hence, the actual output rates of the previous period, the present technique of sector Ⅰ and the current real wage rate jointly determine the equilibrium value of aggregate output in the current period.

1. Determinants of Equilibrium Employment

At equilibrium, what conditions would prevail in the labor market? By assumption, there will be a certain number of unemployed workers. The proportion of the labor force which would remain unemployed, however, depends upon the size of the labor force, the total, the total value of variable capital advanced by capitalists, the value of a consumer good(i.e., λ_2), and the real wage rate. In general

$$L = \frac{V}{\lambda_2(b/T)} \tag{25}$$

If equilibrium prevails in the current period, we know more specifically that

$$L' = \frac{x_2^0}{b/T} \tag{26}$$

since variable capital equals the total value of consumer goods production of the previous period. The total number of employed workers, then, can easily be found by taking into account the customary length of the workday, T :

$$N' = (L'/T) = (x_2^0/b). \tag{27}$$

It is notable that the aggregate hours of employment depend on both the subsistence daily wage and the duration of the work day, whereas the number of employed depends just of the daily real wage. This reflects the assumption that each employed worker must be paid enough money wages to purchase the daily bundle of consumer goods required to reproduce his or her laboring capacities, regardless of the hours worked daily by that worker.

2. Determinants of Value Magnitudes

If equilibrium prevails in the current period, precisely how much surplus value is created during production? The answer to this question is momentous since the total mass of surplus value is a crucial determinant of the total value of capital($C+V$) which capitalists might advance at the beginning of the next period of production.

In general, total surplus value can be calculated as

$$S = Y - K = L - V. \tag{28}$$

Given (20) and (27), however, the equilibrium mass of surplus value is

$$S' = [(T/b) - \lambda_2] \cdot x_2^0. \tag{29}$$

Hence, total surplus value depends upon the value of wage goods(λ_2), the real wage rate and the physical supply of wage goods available for consumption by workers and their dependents.

Under what set of circumstances does exploitation of labor take place? Positive surplus value is produced by employed workers and appropriated by the owners of capital if and only if $(T+b) > \lambda_2$. That is, the number of hours worked per wage good currently consumed must exceed the number of fours of labor time presently required to reproduce each wage good being consumed.[10)]

The value rate of profit (π) can be expressed in general terms as

$$\pi = \frac{(S/V)}{(C/V)+1} \tag{30}$$

10) For a more elegant proof of this theorem, see Morishima(1973, ch. 5).

Where the numerator is the rate of surplus value and where(C/V) varies directly with the organic composition of capital, namely(C/K). In equilibrium, the value rate of profit takes of the specific magnitude.

$$\pi' = \left[\frac{(T/b)}{\lambda_2} - 1\right] / \left[\frac{(\lambda_1/\lambda_2)}{(x_2^0/x_1^0)} + 1\right] \quad (31)$$

Hence, given the organic composition of capital, the value rate of profit varies directly with the length of the working day and inversely with the subsistence real wage per period and value of a wage good.

3. Determinants of Environmental Quality

As this process of the reproduction of commodities unfolds, what happens to environmental quality? That depends of the propensities of sectoral techniques to produe waste emissions and of durability of those emissions, of the scale of production in the current period, and on the concentration of pollutants inherited from earlier periods of history.

$$R' = g_1 x_1^0 + \left[r_3 + \frac{g_2}{(b/T)}\right] \cdot x_2^0 \quad (32)$$

where

$\gamma \equiv (\gamma_1/\alpha_1)/(\gamma_2/\alpha_2)$

$g_1 \equiv [(\gamma_2/\alpha_2)\cdot(\gamma - \alpha)]/(1-\alpha)$, and

$g_2 \equiv [((\gamma_2/l_2)\cdot(1-\gamma)]/(1-\alpha)$.

Hence, the broader conception of technology which we mentioned at the outset is important since one needs to know the sectoral propensities to emit pollutants in order to calculate the aggregate flow of waste matter and energy into the environment.

This current flow of waste emissions is not an accurate physical measure of environmental quality, however, since many waste emissions persist in their toxic forms well beyond their period of discharge and hence accumulate as environmental stocks of pollutants. Nuclear power reactors in the US, for example, routinely discharge small quantities of a wide variety of radio-isotopes. Since many of these pollutants require decades or even centuries to undergo completeradioactive decay, they can induce cancers or genetic mutations long after their date of production and emission into the environment.[11)]

As Nordhaus (1977, p.341) has pointed out, the combustion of fossil fuels leads to the environmental accumulation of another pollutant, in that case carbon dioxide. Although not immediately toxic, its atmospheric "residence time appears to be very long, with approximately one-half of all industrial earbon dioxide still airborne". Some climatologists speculate that continued growth in its atmospheric concentration would eventually lead to substantial climatic changes around the world. Hence, in measuring the present degree of environmental quality, one must take account of both current waste emissions and the portion of earlier discharges which at present remain in costly forms :

$$P' = g_1 x_1^0 + \left[r_3 + \frac{g_2}{(b/T)} \right] \cdot x_2^0 + (1-\delta) \cdot P^0 \qquad (33)$$

Ⅴ. A Potential Contradiction of Capitalist Accumulation

What implications do differences in the rate of surplus value and the rate of profit have for environment quality? That depends upon how such differences come about. A longer working day, for example, means that the same number of employed workers labors a larger total number of hours, produce more aggregate output, and also produce a larger mass of surplus value, which in turn permits a higher rate of profit and encourages faster accumulation of capital. Such a state of affairs would certainly please capitalists, but notice an ominous environmental consequence. This immediate acceleration of capital accumulation would be accompained by a larger flow of waste emissions and consequently lower degree of environmental quality.[12)] To the extent that these

11) For a recent discussion of the social costs of the US nuclear power programme, see England(1979).

additional waste emissions are persistent, their ecological effects would be felt for years to come, not just in the current period.

Although the case of a longer working day is theoretically interesting, it is far less germane in the modern era of capitalist accumulation than that of technical innovation. Knowing the precise character of technical change requires concrete historical research on the techniques which capitalists have introduced into the labor process.

Let us assume, provisionally, however, that technical change is strictly labor-saving and general, i.e., that$(\triangle l_1/l_1) = (\triangle l_2/l_2) < 0$, whereas $(\triangle a_i/a_i) = 0$, $i = 1, 2$.

Since employment conditions are determined by the real wage rate and by the inherited supply of wage goods, there would be no impact on employment during the period when such innovations were introduced. However, technical change of a generally labor saving character would reduce the values of both commodities proportionally and hence would increase both the total mass of surplus value and the rate of profit, thereby acceleration capital accumulation.[13)]

Even if it were true that these new techniques did not require more means of production per unit of gross output in a general sense, they probably would require changes in the physical composition of the materials and energy sources which comprised those means of production. For instance, chemical fertilizers and pesticides might replace manure and natural pest controls in agriculture, whereas nuclear energy and fossil fuels might substitute for water wheels and wind mills as power sources. Commoner, Corr and Stamler(1971, p. 16) have hypothesised that physical substitutions of this sort have been the major cause of declining environmental quality in recent decades:

> "[The] rapid intensification of pollution in the United States in the period 1946-1968 cannot be accounted for by concurrent increases either in population or in affluence(as measured by real GNP per head). What seems to be far more important...is the nature of the...new technologies introduced following World WarⅡ..."

In other words, many of the particular materials and energy sources introduced by capitalists in recent decades to bolster labor productivity have resulted in increased toxicity and environmental

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12) These conclusion result form insertion $\triangle T > 0$ into (24), (26), (27), (29), and (31)-(33) while assuming that the subsistence daily wage and technical parameters remain the same.

13) Sloving the two value equations of (9) simultaneously, one finds that $\lambda_1 = l_1/(1-a_1)$, $\lambda_2 = [a_2 l_1 + (1-a_1)l_2] / (1-a_1)$, and hence $(\lambda_2/\lambda_1) = [a_2 + (1-a_1)(l_2 l_1)]$. Hence generally labor-saving innovation leaves (λ_2/λ_1) unchanged since (l_2/l_1) remains the same and, via (29) and (31), leads to $\triangle S > 0$ and $\triangle \pi > 0$.

durability of waste discharges. In the language of this model, that means $\triangle \gamma_i > 0$, $i = 1, 2, 3$ and $\triangle \sigma < 0$. To the degree that this characterization of technical change is accurate, one can conclude that innovation spurs labor productivity, the rate of profit, and accumulation of capital in the short run, but with the ominous, longer-term consequence that environmental stocks of toxic pollutants accumulate far more-rapidly.

If environmental pollutants were simply an unpleasant nuisance or "disamenity" then this decline of environmental quality would not affect the accumulation process itself. However, there is mounting empirical evidence that pollution adversely affects human health and mortality as well. V. K. Smith(1976, p.60) reports, for example :

> "A review of several representative studies of the air pollution-health relationship....does lend some rather strong support[to the thesis]....that air pollution does have a positive impact on mortality rates."

In a similar vein, the United States Council on Environmental Quality(1975, p.vii) concluded :

> "[Each} year thousands [of additional chemicals]...are discovered by the United States chemical industry and hundreds are introduced commercially ... Many are not toxic, but the sheer number of chemical compounds, [and] the diversity of their use...make it increasingly problem that chemical contaminants in our environment have become a significant determinant of human health and life expectancy."

Although capitalists are certainly exposed to pollutants, industrial and agricultural workers are especially likely to suffer chronic exposure to heavy doses of these contaminants. The cancers, respiratory diseases, birth defects, and other pollution-induced illnesses which result from this chronic exposure frequently surface years or even decades later(Kneese and Schulze, 1977, p.327). Hence the "productivity-enhancing" character of any particular capitalist innovation is pro blematic. In the short run, a new technique might be labor-saving but in the long run it might substantially increase the quantity of medical care required to reproduce workers' laboring capacities.

We therefore arrive at an apparent contradiction of modern capitalist accumulation. To the extent that capitalists have introduced and initiated labor-saving techniques in recent decades, short-term gains in labor productivity, profitability and accumulation of capital have occurred. However, to the extent that these same innovations have produced growing amounts of toxic, persistent pollutants, they will eventually lead to serious, persistent health problems for the working class.

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<국문초록>

재생산에 관한 마르크스의 모형에 있어서 환경의 질에 관한 연구

심 경 섭 *

본 논문은 마르크스의 재생산에 관한 이론으로써 환경문제인 환경의 질과 연관시켜서 2부문의 모형을 도출해 보는 것이다. 따라서 본 논문에서는 최저실질임금, 고정생산요소, 고용될 수 있는 고용자들의 잉여, 자본과 노동에 대한 극대효율화, 환경오염의 수준, 그리고 대량의 잉여가치 창출을 전제로 하여 모형을 도출해 보는 것이다. 또한 이 논문에서의 사용되는 가설은 환경의 질에 관한 최고의 고도화된 기술혁신의 기법을 전제로 하여 설정된 것이다.

* 단국대학교 경제학부 교수, 경제학박사