## Permanent and Temporary Jobs in the Labor Market

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#### Abstract

I ABSTRACT This paper develops a model of search in the two-sector labor market to take the role for temporary jobs in the labor market into account. Steady state analysis of a decentralized equilibrium of the model yields many results well matched with our intuition but have not been highlighted in the literature. For example, a rise in the destruction rate of permanent jobs makes tighter not only the permanent job market but also the temporary job. Moreover, it could enlarge the relative size of the permanent job market if creating a permanent job vacancy is not so much relatively expensive. It is also found that an increase in unemployment benefits could enlarge the relative size of the permanent job market if worker's bargaining power is strong enough.


Key Words : temporary jobs, specialization, unemployment, search equilibrium, labor market

## I. Introduction

Once the labor market is viewed as a set of decentralized locations where workers and entrepreneurs meet to create mutually beneficial jobs, it looks quite natural to assume that creation of a highly productive job requires relatively strict job specific qualifications on both sides of the market. For its creation, usually entrepreneurs should prepare highly specialized workshops, which must be filled with highly specialized workers to become productive. Thus it is natural to consider a labor market matching

[^0]model in which there exist different types of jobs each is indexed by a particular level of productivity positively correlated with the level of its matching difficulty. If an agent wants to form a productive match, he should first choose a target type before searching his counterparty. However, when we look over the entire labor market, it is also natural to assume that there exist low productivity jobs which require little job specific qualification. For an agent who wants to form a match that yields this kind of jobs, it is unnecessary to choose a target type.

To investigate the role for temporary jobs in the labor market, this paper establishes a model of search in the labor market with two sectors: one is for permanent jobs and the other is temporary ones. In the model, each agent's target type in the permanent job market is exogenously given. Clearly, the optimal choice of a permanent job type, which makes the distribution of the agents over permanent job types as an equilibrium outcome, could be an interesting issue associated with endogenous specialization in human capital as well as physical capital. However, this paper focuses on participation in one of two sectors rather than specialization for permanent jobs.

Even though the model is not explicit about the second stage matching in the permanent job market, it is important to recognize that there is backward induction logic behind this simplification. Because both of the workers and entrepreneurs distributions over permanent job types are outcomes from this second stage, they should be the same. The permanent job type of each worker is characterized by the probability that match is 'good', that is, the probability that double coincidence of job specific qualifications happens after he meets a job vacancy in the permanent job market. Of course, this probability is negatively correlated with his permanent job productivity. Anyhow, if the permanent job types, or the matching probabilities in the permanent job market, of all the workers are identical, it can be shown that there is a continuum of decentralized equilibria indexed by the different proportions of permanent job search workers to total unemployed workers. Hence, in order to make the proportion uniquely determined, here it is assumed that the distribution of the workers over permanent job types is nondegenerate.

The main purpose of this paper is to show how the proportion of permanent job search workers to total unemployed workers is determined by the model parameters.

Notice that this proportion directly shows which types of permanent jobs can actually exist in the labor market. This paper also investigates the effects of changes in the model parameters on other statistics of the labor market. Especially, it analyses how changes in the parameters of one sector affect the other sector.

The rest of the paper is organized as follows. The next section develops the model and defines a steady state decentralized equilibrium. Section 3 presents the solution to the model and the qualitative results from comparative static analyses. Section 4 concludes.

## II. Model

## 1. Primitives

The model in this paper is an extension of the standard matching model of search in the labor market. See Mortensen and Pissarides (1999) and Andolfatto (2008) and their references for reviews of the standard matching model. Here, instead of writing down all of the common primitives, let me emphasize the distinguished features of the model in this paper.

First of all, our model has two sectors, called permanent and temporary job sector respectively. They are indexed by $s \in\{H, L\}$, where $H$ indicates the permanent job sector and $L$ indicates the other.

There exists a continuum of workers, with total mass equal to one. A key feature of the model is that heterogeneous workers are distributed over permanent job types indexed by $\varepsilon$. The distribution function of $\varepsilon$ is denoted by $F$. Each unemployed workers can search a job only in one sector in each period. Hence the value of unemployment of a worker with specific permanent job type $\varepsilon$, called type $\varepsilon$ worker henceforth, is given by

$$
U(\varepsilon)=\max \left\{U_{H}(\varepsilon), U_{L}\right\},
$$

where $U_{s}$ represents the value of unemployed search in sector $s$. There also exist a
continuum of entrepreneurs, with total mass equal to one, and they are also distributed over permanent job types. Because the second stage matching within the permanent sector is implicitly assumed, by backward induction, their distribution is assumed to be the same to that of workers. Entrepreneurs can create new job vacancies, that is, create vacant work sites and post them, in any of both sectors as much as they want by paying costs. Here, the value of a new vacancy posted by a entrepreneur with specific permanent job type $\varepsilon$, called type $\varepsilon$ entrepreneur henceforth, is

$$
V(\varepsilon)=\max \left\{V_{H}(\varepsilon), V_{L}\right\},
$$

where $V_{s}$ indicates the value of a vacancy posted in sector $s$.
A standard matching function $m$, which is increasing, concave and CRS, represents the matching technology. Let $v_{s}$ and $u_{s}$ be the masses of job vacancies posted and unemployed workers searching jobs in sector $s$. Then, $m\left(u_{s}, v_{s}\right)$ equals in value to the mass of 'meeting' in sector $s$. Hence the probability that a worker meets a vacancy, called job arrival rate, becomes

$$
\begin{equation*}
\alpha_{w}\left(\eta_{s}\right)=\frac{m\left(u_{s}, v_{s}\right)}{u_{s}}=m\left(1, \eta_{s}\right), \tag{1}
\end{equation*}
$$

where $\eta_{s}=\frac{v_{s}}{u_{s}}$ indicates market tightness (as commonly noted, but actually market slackness for the workers) in sector $s$. At the same time, for an entrepreneur, the arrival rate of a worker is given by

$$
\begin{equation*}
\alpha_{e}\left(\eta_{s}\right)=m\left(\frac{1}{\eta_{s}}, 1\right)=\frac{\alpha_{w}\left(\eta_{s}\right)}{\eta_{s}} . \tag{2}
\end{equation*}
$$

It is assumed that temporary jobs do not require any qualification. Hence, whenever a worker meets a job vacancy in the temporary job sector, they form a productive match, that is, whenever a vacancy is filled with a worker, it becomes productive. The above probabilities are the same to the probabilities that they form a productive match in the temporary job market. In contrast, there exists the problem of double coincidence of job specific qualifications in the permanent job sector. It is assumed that
a match is productive only with probability $\varepsilon$ when a type $\varepsilon$ worker meets an arbitrary job vacancy in the permanent job market. Hence the probability that a type $\varepsilon$ worker forms a match is given by $\varepsilon \alpha_{w}\left(\eta_{H}\right)$. Symmetrically, the probability that a permanent job vacancy posted by a type $\varepsilon$ entrepreneur, called type $\varepsilon$ permanent job vacancy henceforth, forms a match is given by $\varepsilon \alpha_{e}\left(\eta_{H}\right)$.

Every match in the temporary jobs sector produces $y_{L}$ in each period. In the permanent job sector, a match of a type $\varepsilon$ worker and a type $\varepsilon$ vacancy produces $y_{H}(1-\varepsilon)$, where $y_{H}^{\prime}>0$. This reflects the fact that a permanent job with more difficulty in matching have higher productivity once matched.

The discount factor in time preferences is $\beta \in(0,1)$. The unemployment benefit is $b$ in each period. The exogenous job destruction rate in sector $s$ is $\lambda_{s} \in(0,1)$ The cost of creating and maintaining a vacancy in sector $s$ for one period is $k_{s}$. The worker's bargaining power in a bilateral Nash bargaining for the wage determination is $\theta$. The proportion of permanent job search workers to total unemployed workers is denoted by $\pi=\frac{u_{H}}{u}$. For notational simplicity, let $\pi_{H}=\pi$ and $\pi_{L}=1-\pi$.

## 2. Value Functions and Decision Rules

In the temporary sector, Bellman's equations are the same to those in the basic labor market matching model. Given an arbitrary wage $w_{L}$, the associate value of a filled temporary job to the employer, $J_{L}$, solves Bellman's equation:

$$
\begin{equation*}
(1-\beta) J_{L}=y_{L}-w_{L}-\beta \lambda_{L} J_{L} . \tag{3}
\end{equation*}
$$

Given worker arrival rate $\alpha_{e}^{L}$, the value of a new temporary job vacancy is

$$
\begin{equation*}
(1-\beta) V_{L}=-k_{L}+\beta \alpha_{e}^{L} J_{L} \tag{4}
\end{equation*}
$$

Given an arbitrary wage $w_{L}$, the value of a temporary job to the worker, $W_{L}$, satisfies

$$
\begin{equation*}
(1-\beta) W_{L}=w_{L}+\beta \lambda_{L}\left[U(\varepsilon)-W_{L}\right], \tag{5}
\end{equation*}
$$

and the value of unemployed search for a temporary job is

$$
\begin{equation*}
(1-\beta) U_{L}=b+\beta \alpha_{w}^{L}\left[W_{L}-U_{L}\right] \tag{6}
\end{equation*}
$$

Here, it is of use to notice that, in equation (5), $U(\varepsilon)=U_{L}$ since $\varepsilon$ is time invariant. Each worker searches a job in the same sector whenever unemployed.

The value of a type $\varepsilon$ permanent job match to the employer, $J_{H}(\varepsilon)$, solves Bellman's equation:

$$
\begin{equation*}
(1-\beta) J_{H}(\varepsilon)=y_{H}(1-\varepsilon)-w_{H}(\varepsilon)-\beta \lambda_{H} J_{H}(\varepsilon), \tag{7}
\end{equation*}
$$

and the value of a type $\varepsilon$ permanent job vacancy is

$$
\begin{equation*}
(1-\beta) V_{H}(\varepsilon)=-k_{H}+\beta \varepsilon \alpha_{e}^{H} J_{H}(\varepsilon), \tag{8}
\end{equation*}
$$

The value of a permanent job to the type $\varepsilon$ worker, $W_{H}(\varepsilon)$, satisfies

$$
\begin{equation*}
(1-\beta) W_{H}(\varepsilon)=w_{H}(\varepsilon)+\beta \lambda_{H}\left[U_{H}(\varepsilon)-W_{H}(\varepsilon)\right], \tag{9}
\end{equation*}
$$

and the value of unemployed search for a permanent job to the type $\varepsilon$ worker is

$$
\begin{equation*}
(1-\beta) U_{H}(\varepsilon)=b+\beta \varepsilon \alpha_{w}^{H}\left[W_{H}(\varepsilon)-U_{H}(\varepsilon)\right] . \tag{10}
\end{equation*}
$$

Now it can be shown that worker's optimal sector choice is of the critical value form. Let $\widetilde{\varepsilon}$ be this critical value such that

$$
\begin{equation*}
\widetilde{\varepsilon} \alpha_{w}^{H}\left[W_{H}(\widetilde{\varepsilon})-U_{H}(\widetilde{\varepsilon})\right]=\alpha_{w}^{L}\left[W_{L}-U_{L}\right] . \tag{11}
\end{equation*}
$$

Then,

$$
U(\varepsilon)=\left\{\begin{array}{l}
U_{H}(\varepsilon) \text { if } \varepsilon \leq \widetilde{\varepsilon}  \tag{12}\\
U_{L} \text { otherwise }
\end{array}\right.
$$

This is trivial from equation (6) and (10). An unemployed worker search a permanent job if and only if his type $\varepsilon$ is greater than, or equal to, the critical value $\widetilde{\varepsilon}$.

If positive amount of vacancies are posted in one sector, profit maximization and free entry require that all rents from new vacancy posting in that sector are exhausted. Entrepreneurs' decision is characterized by the vacancy posting condition:

$$
\begin{align*}
& V_{L} \leq 0,=0 \text { if } v_{L}>0, \\
& V_{H}(\varepsilon) \leq 0,=0 \text { if } v_{H}>0 \text { and } \varepsilon \geq \widetilde{\varepsilon} . \tag{1}
\end{align*}
$$

## 3. Steady State Decentralized Equilibrium

In each sector, we seek the generalized Nash bargaining wage outcome:

$$
w=\operatorname{argmax}(W-U)^{\theta}(J-V)^{1-\theta} .
$$

This leads the well-known result that the worker's share of match surplus is the constant $\theta$.

$$
\begin{align*}
& W_{L}-U_{L}=\theta\left[J_{L}+W_{L}-V_{L}-U_{L}\right], \\
& W_{H}(\varepsilon)-U_{H}(\varepsilon)=\theta\left[J_{H}(\varepsilon)+W_{H}(\varepsilon)-V_{H}(\varepsilon)-U_{H}(\varepsilon)\right], \forall \varepsilon \geq \widetilde{\varepsilon} . \tag{1}
\end{align*}
$$

Wages are determined in the way satisfying the above equations.
Aggregate state $\pi$ is determined by worker's individual decisions (12) and the distribution of $\varepsilon$ over them:

$$
\begin{equation*}
\pi=F(\widetilde{\varepsilon}) . \tag{15}
\end{equation*}
$$

The arrival rates to individual agents are given by the market tightness and the matching technology:

$$
\begin{equation*}
\alpha_{w}^{s}=\alpha_{w}\left(\eta_{s}\right), \alpha_{e}^{s}=\alpha_{e}\left(\eta_{s}\right), \forall s, \tag{16}
\end{equation*}
$$

where $\alpha_{w}\left(\eta_{s}\right)$ and $\alpha_{e}\left(\eta_{s}\right)$ are given in (1) and (2) respectively. Notice that, at a steady state,

$$
\begin{align*}
& \alpha_{w}\left(\eta_{L}\right)(1-\pi) u-\lambda_{L} e_{L}=0,  \tag{17}\\
& \sigma(\widetilde{\varepsilon}) \alpha_{w}\left(\eta_{H}\right) \pi u-\lambda_{H} e_{H}=0, \tag{18}
\end{align*}
$$

where $\sigma(\widetilde{\varepsilon})=\int_{\tilde{\varepsilon}}^{1} \delta d F(\varepsilon)$. Hence, we have

$$
\begin{equation*}
u=\frac{\lambda_{H} \lambda_{L}}{\lambda_{H} \lambda_{L}+\lambda_{L} \pi \sigma(\tilde{\varepsilon}) \alpha_{w}\left(\eta_{H}\right)+\lambda_{H}(1-\pi) \alpha_{w}\left(\eta_{L}\right)} . \tag{19}
\end{equation*}
$$

Definition. A steady state decentralized equilibrium is a list of the value functions $J_{s}, V_{s}, W_{s}, U_{s}, \forall s \in\{H, L\}$, the unemployment rate $u$, the permanent sector participation ratio $\pi$, the vacancy rates $v_{s}, \forall s$, the wages $w_{s}, \forall s$ and the arrival rates $\alpha_{w}^{s}, \alpha_{e}^{s}, \forall s$ which satisfies; (i) value functions: (3), (4), $\cdots$, (10); (ii) decision rules: (12), (13); (iii) wage determination: (14); and (iv) consistency: (15), (16), (19).

## III. Solution and Qualitative Results

## 1. Model Solution

Let define the net surplus of a match by

$$
\begin{aligned}
& S_{L}=J_{L}+W_{L}-V_{L}-U_{L}, \\
& S_{H}(\varepsilon)=J_{H}(\varepsilon)+W_{H}(\varepsilon)-V_{H}(\varepsilon)-U_{H}(\varepsilon), \forall \varepsilon \geq \widetilde{\varepsilon} .
\end{aligned}
$$

Since we are interested in equilibrium with $v_{s}>0, \forall s$, by summing up the Bellman's equations and using the wage determination rules, we have

$$
\begin{align*}
& (1-\beta) S_{L}=y_{L}-b-\beta \lambda_{L} S_{L}-\beta \alpha_{w}\left(\eta_{L}\right) \theta S_{L}  \tag{20}\\
& (1-\beta) S_{H}(\varepsilon)=y_{H}(1-\varepsilon)-b-\beta \lambda_{H} S_{H}(\varepsilon)-\beta \varepsilon \alpha_{w}\left(\eta_{H}\right) \theta S_{H}(\varepsilon), \forall \varepsilon \geq \widetilde{\varepsilon}, \tag{21}
\end{align*}
$$

and by using the vacancy posting conditions and the wage determination rules, we get

$$
\begin{align*}
& k_{L}=\beta \frac{\alpha_{w}\left(\eta_{L}\right)}{\eta_{L}}(1-\theta) S_{L},  \tag{22}\\
& k_{H}=\beta \varepsilon \frac{\alpha_{w}\left(\eta_{H}\right)}{\eta_{H}}(1-\theta) S_{H}(\varepsilon), \forall \varepsilon \geq \widetilde{\varepsilon} . \tag{23}
\end{align*}
$$

In the temporary job sector, as is standard, $\eta_{L}=\frac{v_{L}}{(1-\pi) u}$ and $S_{L}$ are uniquely determine by solving (20) and (22) simultaneously. Meanwhile, the definition of critical value $\widetilde{\varepsilon}$ in (11) and the wage determination rules yields

$$
\begin{equation*}
\widetilde{\varepsilon} \alpha_{w}\left(\eta_{H}\right) S_{H}(\widetilde{\varepsilon})=\alpha_{w}\left(\eta_{L}\right) S_{L}, \tag{24}
\end{equation*}
$$

and from (22) and (23),

$$
\begin{equation*}
\frac{\varepsilon}{k_{H}} \frac{\alpha_{w}\left(\eta_{H}\right)}{\eta_{H}} S_{H}(\varepsilon)=\frac{\widetilde{\varepsilon}}{k_{H}} \frac{\alpha_{w}\left(\eta_{H}\right)}{\eta_{H}} S_{H}(\widetilde{\varepsilon})=\frac{1}{k_{L}} \frac{\alpha_{w}\left(\eta_{L}\right)}{\eta_{L}} S_{L}, \forall \varepsilon \geq \widetilde{\varepsilon} . \tag{25}
\end{equation*}
$$

Then, by dividing (24) by (25), we have

$$
\begin{align*}
& \eta_{H}=\frac{k_{L}}{k_{H}} \eta_{L}=\frac{1}{\kappa} \eta_{L},  \tag{26}\\
& S_{H}(\widetilde{\varepsilon})=\frac{\alpha_{w}\left(\eta_{L}\right)}{\widetilde{\varepsilon} \alpha_{w}\left(\frac{\eta_{L}}{\kappa}\right)} S_{L}
\end{align*}
$$

Since we knows the values of $\eta_{L}$ and $S_{L}$, by plugging these $\eta_{H}$ and $S_{H}(\widetilde{\varepsilon})$ into (21), we can get the value of $\widetilde{\varepsilon}$. That is, $\widetilde{\varepsilon}$ is a solution to

$$
\begin{equation*}
G(\widetilde{\varepsilon})=y_{H}(1-\widetilde{\varepsilon})-b-\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\alpha_{w}\left(\eta_{L}\right)}{\widetilde{\widetilde{\varepsilon}} \alpha_{w}\left(\kappa \eta_{L}\right)} S_{L}-\beta \theta \alpha_{w}\left(\eta_{L}\right) S_{L}=0 . \tag{27}
\end{equation*}
$$

Thus, if the above equation has the unique solution in $(0,1)$, we can get $\pi$ directly from $\pi=F(\widetilde{\varepsilon})$. To guarantee this unique existence, we assume that $y_{H}(0)$, the minimum permanent job productivity, is small enough to make $G(1)<0$, and that $y_{H}^{\prime}$ is large enough to make $G^{\prime}<0$ on $(0,1)$. In Appendix 1 , it is shown that there exist more arrogant assumptions for this unique existence. Nevertheless, from those assumptions, we have $G^{\prime}>0$ around its solution as shown in the Appendix. Since a change in $G$ has the direction opposite to that in the permanent job productivity $-y_{H}^{\prime}<0$, this leads comparative statistics inconsistent with our intuition. Hence it would be better not to adopt those assumptions.

Once $\pi$ is recovered, we can get the unemployment rate $u$ from (19). Then, from the known values of $\eta_{s}, \forall s$, we can find $v_{s}>0, \forall s$. We are done at this point. However, if we are interested in the sectoral employment rate, we can compute them from (18) and (17).

## 2. Comparative Statistics

In the previous subsection, it is shown that we can solve the model numerically once the model parameters including the functional forms are calibrated. In this subsection, try to find some qualitative results without actually computing the numerical solution.

In each sector, we have the comparative statistics same to those in the basic matching model: $\frac{\partial S_{L}}{\partial b}<0, \frac{\partial S_{H}(\varepsilon)}{\partial b}<0, \forall \varepsilon \geq \widetilde{\varepsilon}, \frac{\partial \eta_{s}}{\partial b}<0, \forall s$ and so on. See Appendix 2 for the complete analysis.

From these comparative statistics in each sector, equation (26) directly shows the effects of parameters in one sector on the market tightness in the other sector. That is, $\frac{\partial \eta_{H}}{\partial y_{L}}>0, \frac{\partial \eta_{L}}{\partial \lambda_{H}}<0$ and so on. It is interesting that the effects of the parameters in the same sector and those in the other sector on the market tightness have the same direction.

Here, for simplicity, let us assume that the matching function is of the form of CobbDouglas, that is, $m(u, v)=u^{1-\gamma} v^{\gamma}, \gamma \in(0,1)$. Then, equation (25) becomes

$$
\varepsilon S_{H}(\varepsilon)=\kappa^{\gamma} S_{L}, \forall \varepsilon \geq \widetilde{\varepsilon} .
$$

This equation links changes in the parameters in one sector with changes in the match surplus on the other sector. Now, equation (27) becomes

$$
G(\widetilde{\varepsilon})=y_{H}(1-\widetilde{\varepsilon})-b-\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} S_{L}-\beta \theta \alpha_{w}\left(\eta_{L}\right) S_{L}=0
$$

Notice that, in the previous subsection, it is assumed that

$$
\frac{\partial G}{\partial \widetilde{\varepsilon}}=-y_{H}^{\prime}(1-\widetilde{\varepsilon})+\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} S_{L}<0 .
$$

For any parameter $\varphi$, if we find the sign of $\frac{\partial G}{\partial \varphi}$ from our already known comparative statistics, we can directly get the sign of $\frac{\partial \widetilde{\varepsilon}}{\partial \varphi}$ by the implicit function
theorem. Once it is found, from $\pi=F(\widetilde{\varepsilon})$, we get the sign of $\frac{\partial \pi}{\partial \varphi}$, which yields the key findings in this paper.

First, we consider the effect of a rise in the temporary job productivity $y_{L}$. We have

$$
\frac{\partial G}{\partial y_{L}}=-\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} \frac{\partial S_{L}}{\partial y_{L}}-\beta \theta\left\{\alpha_{w}^{\prime}\left(\eta_{L}\right) \frac{\partial \eta_{L}}{\partial y_{L}} S_{L}+\alpha_{w}\left(\eta_{L}\right) \frac{\partial S_{L}}{\partial y_{L}}\right\}<0
$$

where $\frac{\partial S_{L}}{\partial y_{L}}>0$ and $\frac{\partial \eta_{L}}{\partial y_{L}}>0$. Hence $\frac{\partial \widetilde{\varepsilon}}{\partial y_{L}}<0$ and $\frac{\partial(1-\pi)}{\partial y_{L}}>0$. This result is quite intuitive. We can imagine that more workers become willing to search temporary jobs because those jobs yields wages higher than before.

Next, we consider the effects of the job destruction rates $\lambda_{L}$ and $\lambda_{H}$. For a change in the temporary job destruction rate, we have

$$
\frac{\partial G}{\partial \lambda_{L}}=-\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} \frac{\partial S_{L}}{\partial \lambda_{L}}-\beta \theta\left\{\alpha_{w}^{\prime}\left(\eta_{L}\right) \frac{\partial \eta_{L}}{\partial \lambda_{L}} S_{L}+\alpha_{w}\left(\eta_{L}\right) \frac{\partial S_{L}}{\partial \lambda_{L}}\right\}>0
$$

where $\frac{\partial S_{L}}{\partial \lambda_{L}}<0$ and $\frac{\partial \eta_{L}}{\partial \lambda_{L}}<0$. Hence $\frac{\partial \widetilde{\varepsilon}}{\partial \lambda_{L}}>0$ and $\frac{\partial(1-\pi)}{\partial \lambda_{L}}<0$. This result is also quite intuitive. We can imagine that less workers become willing to search temporary jobs because those jobs do not last so long as before when employed. However, for a change in the permanent job destruction rate, we have

$$
\begin{equation*}
\frac{\partial G}{\partial \lambda_{H}}=-\beta \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} S_{L}-\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} \frac{\partial S_{L}}{\partial \lambda_{H}}-\beta \theta\left\{\alpha_{w}^{\prime}\left(\eta_{L}\right) \frac{\partial \eta_{L}}{\partial \lambda_{H}} S_{L}+\alpha_{w}\left(\eta_{L}\right) \frac{\partial S_{L}}{\partial \lambda_{H}}\right\} \tag{28}
\end{equation*}
$$

where $\frac{\partial S_{L}}{\partial \lambda_{H}}<0$. and $\frac{\partial \eta_{L}}{\partial \lambda_{H}}<0$. Here, the sign of $\frac{\partial G}{\partial \lambda_{H}}$ is ambiguous, and hence, so is the sign of $\frac{\partial \widetilde{\varepsilon}}{\partial \lambda_{H}}$ and $\frac{\partial \pi}{\partial \lambda_{H}}$. Notice that there exist both direct and indirect effects. The first term of the above equation implies that less workers become willing to search permanent jobs because those jobs do not last so long as before when employed. This can be called the direct effect. However, entrepreneurs do not post enough amount of
temporary job vacancies required to totally absorb this movement, that is, $\frac{\partial \eta_{L}}{\partial \lambda_{H}}<0$. Hence, if the temporary job market becomes remarkably more tight than the permanent job market, more workers may search permanent jobs. Of course, if this indirect effect is exceeded by the direct effect, we can say that smaller portion of unemployed workers are willing to search permanent jobs than before when the permanent job destruction rate increases. That is, $\frac{\partial \pi}{\partial \lambda_{H}}<0$. It can be shown that that this happens if $\kappa$, the ratio of the cost of positing a permanent job vacancy to that of posting temporary one, is high enough. We can imagine that the permanent job market cannot become relatively less tight if creating permanent job vacancies requires costs much more than creating temporary ones to entrepreneurs.

We can do the same analysis for other model parameters. Let me consider just one more case of a rise in unemployment benefit $b$. We have

$$
\frac{\partial G}{\partial b}=-1-\left\{1-\beta\left(1-\lambda_{H}\right)\right\} \frac{\kappa^{\gamma}}{\widetilde{\varepsilon}} \frac{\partial S_{L}}{\partial b}-\beta \theta\left\{\alpha_{w}^{\prime}\left(\eta_{L}\right) \frac{\partial \eta_{L}}{\partial b} S_{L}+\alpha_{w}\left(\eta_{L}\right) \frac{\partial S_{L}}{\partial b}\right\},
$$

where $\frac{\partial S_{L}}{\partial b}<0$. and $\frac{\partial \eta_{L}}{\partial b}<0$. The sign of $\frac{\partial G}{\partial b}$ is ambiguous. However, in case that worker's bargaining power $\theta$ is strong enough, then we have $\frac{\partial G}{\partial b}>0$, and hence $\frac{\partial \widetilde{\varepsilon}}{\partial b}>0$ and $\frac{\partial \pi}{\partial b}>0$. We can imagine that, if unemployment benefits increase, less workers are willing to have either permanent or temporary jobs. However, if worker's bargaining power is strong, workers will weight the productivity more than the matching probability. Thus a larger portion of unemployed workers are willing to search permanent jobs.

## IV. Concluding Remarks

This paper develops a model in which the labor market has both permanent and
temporary job sectors. The temporary job market in our model has special characteristic that workers can get jobs whenever they meet job vacancies there, but they have relatively low productivity when employed.

This paper provides sufficient conditions for the model have the unique solution, and the process to get that solution. This will be useful for further quantitative analysis once appropriate model parameterization is done.

It also investigates comparative statistics to find out qualitative results without actually computing that solution. Together with the appropriate assumption on the production technology in the permanent job sector, it obtains the results well matched with our intuition, and these findings highlight the interactions between two sectors in the labor market. For example, a rise in the destruction rate of permanent jobs makes tighter not only the permanent job market but also the temporary job market in a steady state equilibrium. Clearly, this happens since some unemployed workers of low permanent job productivity leaves the permanent job market and moves into the temporary job market. As a result, it could enlarge the relative size of the permanent job market if creating a permanent job vacancy is not so much relatively expensive. It is also noticeable that an increase in unemployment benefits could enlarge the relative size of the permanent job market if worker's bargaining power is strong enough.

## I Appendix

A1.
Define $\quad \varphi=\left\{1-\beta\left(1-\lambda_{H}\right)\left\{\left\{\alpha_{w}\left(\eta_{L}\right) / \alpha_{w}\left(\kappa \eta_{L}\right)\right\} S_{L} \quad\right.\right.$ and $\quad \psi=b+\beta \theta \alpha_{w}\left(\eta_{L}\right) S_{L}, 0<\varphi, \psi<\infty \quad$ for notational simplicity. Then,

$$
G(\widetilde{\varepsilon})=y_{H}(1-\widetilde{\varepsilon})-\frac{\varphi}{\widetilde{\varepsilon}}-\psi .
$$

If $y_{H}(0)$ has a finite value and $y_{H}(1)>\varphi+\psi$, then,

$$
\begin{equation*}
\lim _{\tilde{\varepsilon} \rightarrow 0} G(\widetilde{\varepsilon})>0, G(1)>0 . \tag{A1.1}
\end{equation*}
$$

If $y_{H}$ is continuous, so is $G$. There exist a solution. To show the uniqueness, first note that

$$
G^{\prime \prime}(\widetilde{\varepsilon})=y_{H}^{\prime \prime}(1-\widetilde{\varepsilon})-\frac{2 \varphi}{\widetilde{\varepsilon}^{3}}<0, \forall \widetilde{\varepsilon} \in(0,1),
$$

if $y_{H}$ is linear or concave. So, to satisfy (A1.1), $G$ should increase at least on small enough $\widetilde{\varepsilon}$. If it reaches at its maximum greater than $\varphi+\psi$, then it decreases, but only to $G(1)>0$. The graph of $G$ intersects the $x$-axis only once, and it has a positive slope around that point.

A2.
Suppress the subscript $s \in\{H, L\}$ here. For the permanent job sector, fix $\varepsilon \geq \widetilde{\varepsilon}$, and let $S=\varepsilon S_{H}(\varepsilon)$. Define

$$
Q(S, \eta)=\left[\begin{array}{c}
y-b-\{1-\beta(1-\lambda)\} S-\beta \alpha_{w}(\eta) \theta S \\
-k+\beta \alpha_{e}(\eta)(1-\theta) S
\end{array}\right]
$$

By applying the Implicit Function Theorem, for arbitrary parameters set $\varphi$, we have

$$
\left[\begin{array}{l}
\frac{\partial S}{\partial \varphi} \\
\frac{\partial \eta}{\partial \varphi}
\end{array}\right]=-\left(D_{(s, \eta)} Q\right)^{-1} D_{\varphi} Q
$$

$$
=\frac{1}{\operatorname{det}(Q)}\left[\begin{array}{cc}
-\beta \alpha_{e}^{\prime}(1-\theta) S & -\beta \alpha_{w}^{\prime} \theta S \\
\beta \alpha_{e}(1-\theta) & 1-\beta(1-\lambda)+\beta \alpha_{w}
\end{array}\right] D_{\varphi} Q,
$$

where $\operatorname{det}(Q)=\beta(1-\theta) S\left[\{1-\beta(1-\lambda)\}\left(-\alpha_{e}^{\prime}\right)+\beta \alpha_{e}^{2} \theta\right]>0$. In most cases, it is enough to consider the signs of the above elements.

$$
\left[\begin{array}{l}
\frac{\partial S}{\partial \varphi} \\
\frac{\partial \eta}{\partial \varphi}
\end{array}\right]=(+)\left[\begin{array}{cc}
(+) & (-) \\
(+) & (+)
\end{array}\right] D_{\varphi} Q .
$$

Define $\varphi=(y, b, \lambda, k)$. Then,

$$
D_{\varphi} Q=\left[\begin{array}{cccc}
1 & -1 & -\beta S & 0 \\
0 & 0 & 0 & -1
\end{array}\right] .
$$

It can be easily shown that $\frac{\partial S}{\partial y}>0, \frac{\partial \eta}{\partial y}>0$ and so on. Now, define $\varphi=(\beta, \theta)$. Then,

$$
D_{\varphi} Q=\left[\begin{array}{cc}
-\beta \alpha_{w} S & \left(1-\lambda-\alpha_{w} \theta\right) S \\
-\beta \alpha_{e} S & \alpha_{e}(1-\theta) S
\end{array}\right] .
$$

Once again, it is trivial $\frac{\partial \eta}{\partial \theta}<0$, and we have $\frac{\partial \eta}{\partial \beta}=(+)\left[\left\{1+\beta \alpha_{w}(1-\theta)\right\} \alpha_{e}(1-\theta) S\right]>0$. $\frac{\partial S}{\partial \theta}=(+)\left[\left(\alpha_{e}^{\prime} \alpha_{w}+\alpha_{e}^{2} \theta\right) \beta^{2} S^{2}\right]$ and $\frac{\partial S}{\partial \beta}=(+)\left[-\left\{\alpha_{e}^{\prime}(1-\lambda)+\alpha_{e}^{2} \theta\right\} \beta(1-\theta) S^{2}\right]$ have ambiguous signs.

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# 임시직의 존재를 감안한 노동시장의 탐색균형 분석 

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## I 요 약

본 논문은 2 부문 노동시장 탐색모형을 구축하여 노동시장에서 영구직과 임시직 사이의 상호작용에 대하여 연구하였다. 모형의 정상상태균형에 대한 분석은 직관에 잘 부합하면 서도 기존의 연구들에서는 다루어지지 않았던 결과들을 보여준다. 예를 들어 영구직의 폐 업률이 높아지면 영구직 뿐만 아니라 임시직 노동시장 역시 취업률이 낮아지는 현상이 나 타나는데, 이는 한계수준의 생산성을 가진 구직자들이 영구직 대신 임시직 시장으로 이동 하기 때문에 발생한다. 이 때 만약 기업가들이 임시직을 창출하는 비용이 영구직에 비해 충분히 저렴하지 않다면, 영구직의 폐업률이 높아질 때 노동시장에서 영구직이 차지하는 비중이 오히려 높아질 수도 있는 이론적인 가능성이 나타난다. 이외에도 노동자들의 교섭 력이 충분할 경우, 실업수당의 증가에 따라 노동시장에서 영구직이 차지하는 비중이 높아 지는 결과가 나타나는데, 이는 실업에 따른 순비용이 감소함에 따라 취업하기는 어렵지만 일단 취업에 성공하면 보수가 높은 영구직을 탐색하는 구직자들이 증가하기 때문이다.

핵심주제어 : 임시직, 전문화, 실업, 탐색균형, 노동시장

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