Capital Budgeting and Risk Management

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I. Approaches to Capital Budgeting under Uncertainty

1. Decision Theory Framework of Capital Budgeting

Ever since the capital budgeting problem originally formulated by LORIE and SAVAGE [1955] as well as by DEAN [1956] came to be recognized as a programming model which requires a simultaneous solution of the investment and the financing problems, the extension of these programming models into the realm of uncertainty has been a focus of interest among investment theorists.

Various approaches to solve this problem have been attempted. All of them followed the guidelines of modern decision analysis. This has meant that some form of (μ, σ-)optimization was selected as an objective function to be maximized and that a procedure was developped which achieved this aim.

It is not my intention to deal at length with these approaches here. It may suffice to recall the different model types that try to formulate the capital budgeting problem under uncertainty.

One of the first approaches was to combine the maximization of the expected net present value with the pay-back criterion (ALBACH [1960]). This model originally formulated as a deterministic model with expected net present value as the objective function and the pay-back criterion for the total investment outlay as a constraint, has in the meantime been incorporated in the chance-constrained-programming model (BYRNE, CHARNES, COOPER and KORTANEK [1971]).

Then the early sixties witnessed the discussion between the adherents of two-stage linear programming under uncertainty and the chance-constrained-programmers (DANTZIG and MADANSKY [1961], SENGUPTA, TINTNER and MILLHAM [1963]). Two-stage linear programming under

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uncertainty formulated a model which assumes that whenever a financial constraint is violated in one particular year it is always possible to obtain short-term credit to avoid bankruptcy and that this conditional short-term credit is granted at a particular cost which will then have to be borne by the firm.

This "charge-account approach" contrasts very sharply with the chance-constrained-programming approach. Basically this approach goes on the assumption that the investment planner can preassign the probability with which he wants the financial constraints of the capital budgeting problem to hold in any period within the planning horizon and that he is perfectly willing to risk ruin with the complement of this probability.

A fourth attempt to handle uncertainty in the capital budgeting framework follows the flexible planning approach first advanced by HART [1940]. Flexible planning tries to take into consideration all the possible future course of action that may be taken under all the possible states of the environment that can be foreseen in the present situation (LAUX [1975]). In a recent paper HAX [1976] has shown that the flexible planning approach is consistent with the chance-constrained-programming approach.

We may thus summarize that the chance-constrained-programming approach is the method of capital budgeting under uncertainty which has gained widest acceptance whereas the two-stage-linear-programming approach has not found many followers. This may be mainly due to the fact that the assumptions placed on the banking system by the latter approach seemed to be too rigorous.

2. Risk Management Framework of Capital Budgeting

Capital budgeting under uncertainty can be attacked in a much broader sense than has hitherto been customary if the capital budgeting decision is placed within the framework of risk management.

In the following section I will shortly outline risk management and the role insurance plays in the risk manager's portfolio of instruments and then proceed to formulate a model of capital budgeting under uncertainty in the context of risk management.

II. Risk Management

1. Risk Accounting

Risk management emanates from the insurance management function in the corporation[1]. Insurance managers in industry have traditionally viewed their function quite differently from the insurance seller's function of an insurance firm. They have defined their job in broader terms than just fulfilling the task of buying insurance as cheaply as possible. They have tried to determine the optimal amount of self-insurance and to buy insurance only for the risks in excess of the level to be borne by the company itself. They have also taken active steps to prevent the company from incurring risks[2].

Within recent years risk management has become an integral part of strategic management (RENNIE [1960]). This involves two distinctive functions:

— risk accounting
— risk portfolio planning.

By *risk accounting* I mean that an inventory of all the risks a company is exposed to is kept on a regular basis and that risks are then assessed in a systematic way[3].

If one tries to keep an inventory of all the risks a company may face in the future, one has to determine what a risk is. Two schools of thinking can be distinguished in this context. Those that come from the insurance management side of risk management are trained to consider pure risk only as the object to put in the risk inventory. A pure risk is an event that results in a loss to the company. The typical example of a pure risk is the fire hazard that is inherent in any building. A speculative risk on the other hand involves the risk of a loss as well as the chance of a gain[4]. Those risk

[1] Even today risk management in the academic society is considered part of the insurance field and is taught by professors of insurance. The prominent textworks on risk management, such as the one by MEHR and HEDGES [1974], WILLIAMS JR. and HEINS [1971] and MACDONALD [1966] are authored by professors of insurance and are devoted primarily to managing pure risks by means of insurance.

[2] The early sixties witnessed a spreading notion that risk management had a broader scope than just managing the insurance portfolio. Pioneering publications were e.g.: *American Management Association* [1961], *American Management Association* [1962] and BOLFITT [1964]. However, management science is still very slow in recognizing the potential of risk management for broadening the concepts of the theory of planning under uncertainty.

[3] The problem of risk accounting has found little attention among scholars of risk management. Descriptions of the risk management process normally comprise the three steps risk identification, risk assessment, and risk treatment, ALLEN and DUVALL [1971]

[4] The classification of risks as pure or speculative was introduced by MOWBRAY, cf. MOWBRAY, BLANCHARD and WILLIAMS JR. [1969].

managers that come from the long-range planning side of the business are inclined to consider pure risks as well as speculative risks as the object of risk management. The recent developments in the legal attitude towards product liability[5] have made it apparent that in many cases it is very hard to draw a line of distinction between pure and speculative risks (DENENBERG, FERRARI [1966]). Therefore, I shall consider pure as well as speculative risks as the objects of risk management.

Risk assessment requires an estimation of the maximum possible loss that a company may incur when it invests in certain objects. In a more general sense assessing pure as well as speculative risks means that the whole profit and loss distribution of the risk bearing object has to be estimated.

2. *The Risk Management Portfolio*

After the risks have been identified and assessed a decision has to be reached how to handle them. Risk management distinguishes between

— risk avoidance
— risk transfer
— insurance
— risk spreading
— risk assumption[6].

Risk avoidance has traditionally been considered the task of the engineering function within the firm. Buildings have to be constructed and machines have to be designed so that they can be considered "safe at any speed".

Risk transfer is the task of the legal function within the company. The rules of doing business as well as the contracts a firm signs have to be formulated so that the risk of the business has to be borne by the other parties involved to the greatest possible extent.

The remaining risks have to be *insured*, and the insurance manager has to make sure that he gets adequate coverage at minimum cost to the company. It is evident from the above that he will not come into the picture until the engineering and the legal departments have done their respective jobs.

Of course one can always try to economize on the insurance budget by trying to cope with the risks involved in the business. This could be done either by spreading the risks over a greater number of activities or by as-

[5] Such developments are for example the Consumer Product Safety Act (CPSA) of 1972 in the U.S. and the 2nd Arzneimittelgesetz of 1974 in Germany.

[6] In their treatment of pure risk management most textbooks propose different classifications of risk management devices. Due to the focus on pure risk these devices are oriented towards loss control. HEDGES and MEHR thus distinguish loss retention, loss reduction and loss transfer. MEHR and HEDGES [1974].

suming them. This instrument of *risk spreading* has become to be called diversification in the context of marketing management.

Last but not least, *risk assumption* is a possibility if insurance is too costly or non-existent.

Risk management then means the allocation of the firm's funds among the different risk-handling instruments.

Traditionally this has been done by following a risk strategy: major emphasis is for instance laid on risk avoidance, which of course results in high construction costs and a low insurance budget. Or, to take another example, the risk strategy is heavily in favour of insurance: this then leads to a high insurance budget and risk avoidance at the legally required level on the one hand and low cash reserves for meeting losses on the other.

It is evident from this description that these risk strategies are one-sided and sub-optimal at best.

The simultaneous determination of the optimal level of all the risk management instruments in an optimal risk portfolio is a task that has to be solved.

In the following section we will use this approach to solve the capital budgeting problem under uncertainty.

III. The Optimal Capital Budget under Uncertainty: The Optimal Risk Management Portfolio

1. The Optimal Combination of Risk Avoidance and Risk Assumption

(a) General Formulation

If only risk avoidance and risk assumption are admitted as instruments of risk management in determining the optimal capital budget, then of course the model is that of the traditional formulation of chance-constrained-programming:

(1) $$\max E(C_0) = \bar{c}' x + v' y$$

subject to

(2) $$\bar{B}^* x + D^* y \leqq w^* - R(\pi)$$

(3) $$A x \leqq O$$

(4) $$x \leqq K$$

(5) $$y \leqq L$$

and the non-negativity constraints.

We assume that maximizing the expected value of the net present value of the capital budget under financial constraints which have to hold with probability π is an operational formulation of rational behavior under uncertainty for the decision-maker under consideration[7].

$R(\pi)$ is defined by (6)

$$(6) \qquad R_t(\pi_t) = \lambda_t \left(\sum_{T=0}^{t} \sum_{i=1}^{n} \sigma_{Ti}^2 \bar{g}_{Ti}^2 x_i^2 \right)^{1/2}$$

with λ the deviation in standard units and σ_{Ti} the standard deviation of the cash flows expressed here as a fraction of the expected cash flow $\bar{g}$.

(b) An Example

The example of my book "Investition und Liquidität" is now taken to illustrate this model. We assume that the data of the example are expected

Table 1. Optimal Capital Budget. Risk Avoidance and Risk Assumption

Investment program			Financing program	
Type	Number	Amount	Type	Amount
A(1)	—	—	SF	100.000
A(2)	6.92	484.400	K(1)	—
A(3)	5.58	279.000	K(2)	674.957
B(1)	7.49	823.900	K(3)	1.500.000
B(2)	10.63	531.500	K(4)	733.343
B(3)	10.35	207.000	K(5)	1.000.000
C(1)	—	—	K(6)	—
C(2)	11.84	473.600		
C(3)	45.79	457.900		
Disagio:		200.000		
Cash:		524.000		
Total		3.981.300		3.981.300
Expected net Present value:				
Programs		8.209.037		2.778.817
Capital budget				5.430.220
Cash at the end of Period 1		2.129.853		
Transaction motive		500.692		
Security motive		1.629.161		

[7] We assume normal distributions of the cash flows model and zero costs for the case of violating the constraints. For a more general treatment of the costs of bankruptcy see ALBACH and SCHÜLER [1970].

values and that the standard deviation is treated as a parameter. At first we assume that σ is equal to 90% and that the required level of meeting the financial constraints is 95%.

The optimal capital budget is given in Table 1.

Risk avoidance is clearly evident by the fact that the most profitable machine A(1) is not bought due to the fact that the financial risks involved are too great.

Risk assumption on the other hand takes various forms.

First we notice that risks are spread among several types of machines for each production department of the plant. Secondly 2,129,853 DM are carried in cash at the end of period 1. This amount is partly needed to meet the financial requirements in the following period, but the larger amount of 1,629,161 is kept as the risk reserve $R_2(\pi)$.

2. *The Optimal Combination of Risk Avoidance, Risk Assumption and Insurance*

(a) General Formulation

We now assume that in addition to risk avoidance and risk assumption the firm can take out insurance on any of the capital investment proposals it has at hand. The insurance covers any loss that is incurred by the investment object in any period of its lifetime.

The insurance on a machine costs an annual premium of

$$P_{ti} = w\,r\left\{\sum_{t=1}^{N_i} E(L_{ti})\,q^{-t}\right\} + F \tag{7}$$

where w — annuity factor ($w=(q-1)\,q^{N_i}/(q^{N_i}-1)$)
r — premium factor
L_{ti} — loss of project i incurred in period t
E — expected value
q — discount factor ($q=1+(p/100)$)
F — fix part of the insurance premium
N_i — economic lifetime of project.

The loss function is defined by

$$L_{ti} = \begin{cases} g_{ti} \\ 0 \end{cases} \quad \text{if } g_{ti} \begin{cases} <0 \\ \geq 0 \end{cases} \tag{8}$$

with g_{ti} — the cash flow of object i in period t, a stochastic variable with a normal distribution $f(g_{ti}) = N(\bar{g}_{ti}, \sigma_{ti})$.

Taking out an insurance on the capital investment proposal changes the expected value of the net present value and its standard deviation. Let the net present value of the capital investment proposal without insurance be detired by

$$(9) \qquad E(C_{oi}) = \sum_{t=1}^{N_i} \int_{-\infty}^{+\infty} g_{ti} f(g_{ti})\, dg_{ti}\, q^{-t} + S_{N_i} q^{-N_i} - a_{oi}$$

with N_{Ni} – resale or scrap value of the investment at the end of its economic lifetime

a_{oi} – acquisition cost.

The variance is defined by

$$(10) \qquad \sigma^2(C_{oi}) = \sum_{t=1}^{N_i} \sigma_{ti}^2\, \bar{g}_{ti}^2 .$$

Then the expected net present value of an investment proposal with insurance is given by

$$(11) \qquad E(C^P_{oi}) = \sum_{t=1}^{N_i} \int_{0}^{\infty} g_{ti} f(g_{ti})\, dg_{ti}\, q^{-t} + S_{N_i} q^{-N_i} - a_{oi} - \sum_{t=1}^{N_i} P_{ti} q^{-t}$$

and the variance is defined by

$$(12) \qquad \sigma^2(C^P_{oi}) = \sum_{t=1}^{N_i} \left[\sigma_{ti}^2(g_{ti}) - \sigma_{ti}^2(L_{ti}) + 2\, E(L_{ti}) \int_{0}^{+\infty} g_{ti} f(g_{ti})\, dg_{ti} \right].$$

It can be shown that $\sigma^2(C^P_{oi})$ is always smaller than $\sigma^2(C_{oi})$.

It is evident that the expected net present value of the investment proposals with insurance is always lower than the expected net present value of an investment proposal that is not covered by insurance.

We then formulate the model of capital budgeting with risk avoidance, risk assumption and insurance.

$$(13) \qquad \max E(C_0) = \bar{c}' x + \bar{c}^{P\prime} x^P + v' y$$

subject to

$$(14) \qquad \bar{B}^* x + \bar{B}^{*P} x^P + D^* y \leqq w^* - R^0(\pi)$$

$$(15) \qquad A x + A^P x^P \leqq O$$

$$(16) \qquad x - x^P \leqq K$$

$$(17) \qquad y \leqq L$$

and the non-negativity constraints.

The risk reserve R^0 in (14) is defined by

$$(18) \qquad R_t^0(\pi) = \lambda_t \left[\sum_{T=0}^{t} \sum_{i=1}^{n} \sigma_{Ti}^2 \bar{g}_{Ti}^2 x_i^2 + \sum_{T=0}^{t} \sum_{i=1}^{n} (\sigma_{Ti}^P)^2 \cdot (\bar{g}_{ti}^P)^2 (x_i^P)^2 \right]^{1/2}$$

where $(\sigma_{Ti}^P)^2$ is defined by the expression in the brackets in (12) divided by the expected cash flow of the truncated cash flow distribution function as given in (11).

(b) An Example

We now solve the example given in Section III.1 above with an insurance option added for each investment proposal. The enlarged model requires the solution of a 22 by 24 model as compared to the 22 by 16 model in Section III.1.

For the numerical calculations we set $\sigma = 90\%$ and $\pi = 95\%$ as before and assume that the fix risk premium is DM 200 per year and per object to be insured. The optimal capital budget is given in Table 2.

Table 2. Optimal Capital Budget, Risk Avoidance, Risk Assumption and Insurance

Investment program				Financing program	
Type	Number without insurance	Number with insurance	Total amount	Type	Amount
A(1)	–	–	–	SF	100,000
A(2)	3.64	8.86	875,000	K(1)	–
A(3)	–	–	–	K(2)	1,000,000
B(1)	7.47	9.31	1,845,800	K(3)	1,500,000
B(2)	0.98	1.06	102,000	K(4)	675,578
B(3)	–	–	–	K(5)	1,000,000
C(1)	–	–	–	K(6)	1,000,000
C(2)	5.28	9.07	574,000		
C(3)	26.81	15.97	427,800		
Disagio:			200,000		
Cash:			250,978		
Total amount			4,275,578		4,275,578
Expected net Present value:					
Programs			11,425,866		3,094,266
Capital budget					8,331,600
Cash at the end of Period 1	2,042,275				
Transaction motive	30,015				
Security motive	2,018,260				

Insurance indeed adds a new dimension to capital budgeting under uncertainty: It allows the firm to invest in riskier projects, to invest a larger sum, to reduce liquidity holding before the first period considerably, and to reduce risk spreading among several types of machines in favour of concentration on the highly profitable but risky types of machines.

A comparison of the cash held at the end of the first period for security purposes reveals interesting features of the risk management approach to capital budgeting. The risk reserve is greater absolutely and relatively (with respect to the total sum of the capital budget) in the case of insurance, risk assumption and risk avoidance than in the case of pure risk avoidance. This is plausible because insurance permits the firm to invest in projects with a high contribution to the expected net present value of the capital budget but also with a high contribution to the risk of violating the financial constraints. In other words: insurance allows the firm to shift the structure of its investment program to projects with a longer pay-off period. The cost of insurance, however, induces the firm to pursue this strategy of risk management only partly: risk assumption seems closely connected with insurance. The firm invests more heavily in the risk reserve in the case of insurance than in the case of risk avoidance. It is obvious that a policy of insurance and high liquidity holdings for risk assumption is more profitable than a policy of pure risk avoidance: If we measure the benefits of the different policies by the ratio of net present value of the investment program to the amount of the risk reserve, we find that the benefit of one unit of risk reserve is higher in the case of insurance, risk assumption and risk avoidance than in the case of pure risk avoidance.

In this particular example there is no single type of machine that is completely covered by insurance or completely left without coverage. However, this need not be the case. If we incorporate different variances for the individual machines, then we find that those machines with a low standard deviation are usually left without coverage whereas it seems that it is always possible to assume part of the risk even of the very risky types of investment proposals.

We thus conclude: the risk management approach to capital budgeting is a very definite advance over the decision analysis approach. However: to find the optimal combination of insurance coverage, risk assumption, risk spreading and risk avoidance is not an easy task and requires a very careful balancing of costs involved and the risk inherent in each of the investment proposals and the situation of the company.

(c) Some Extensions

I will now elaborate on the risk management model of capital budgeting by carrying out some sensitivity analyses. Four fundamental parameters of the model will be varied

— the risk inherent in the acquisition proposals
— the fixed insurance premium
— the equity funds available for investment
— the probability requirement for the financial constraints.

(c1) The Optimal Capital Budget under Variable Risk

We assume that the risk involved in the investment proposals is equal for each machine. It is varied from 0 to 120%. For each σ between 0 and

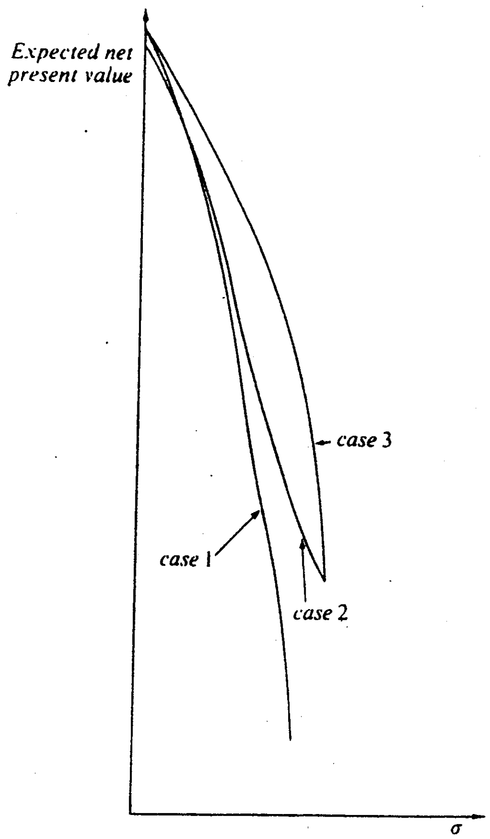

Figure 1

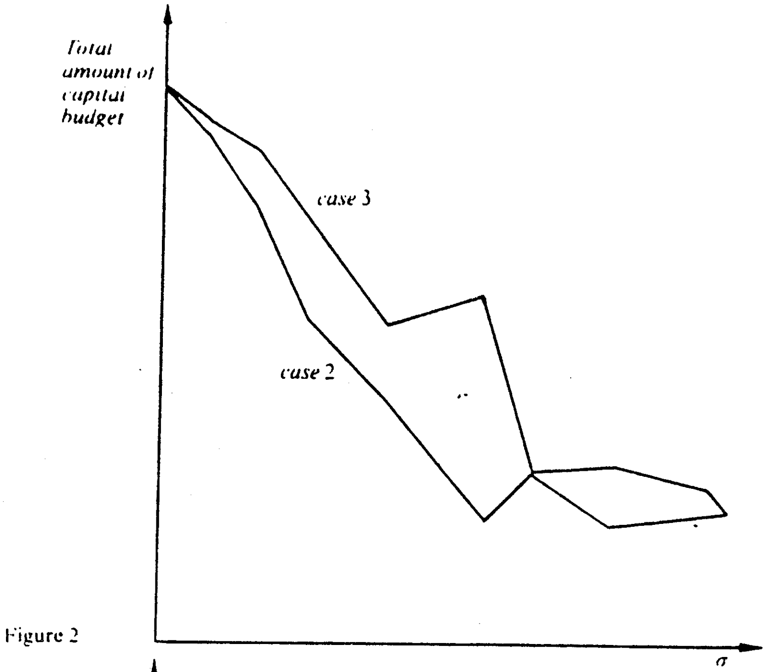

Figure 2

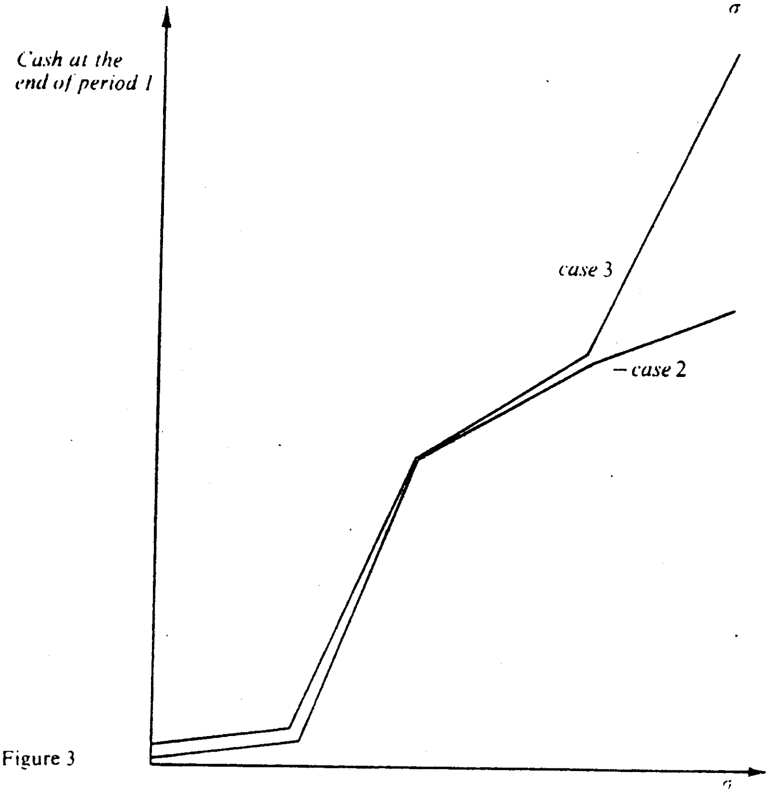

Figure 3

120%, the optimal capital budget is computed. The following characteristic figures are simultaneously determined

— expected net present value of capital budget
— total amount of capital budget
— cash carried over from period 1 to period 2
— annual insurance premium.

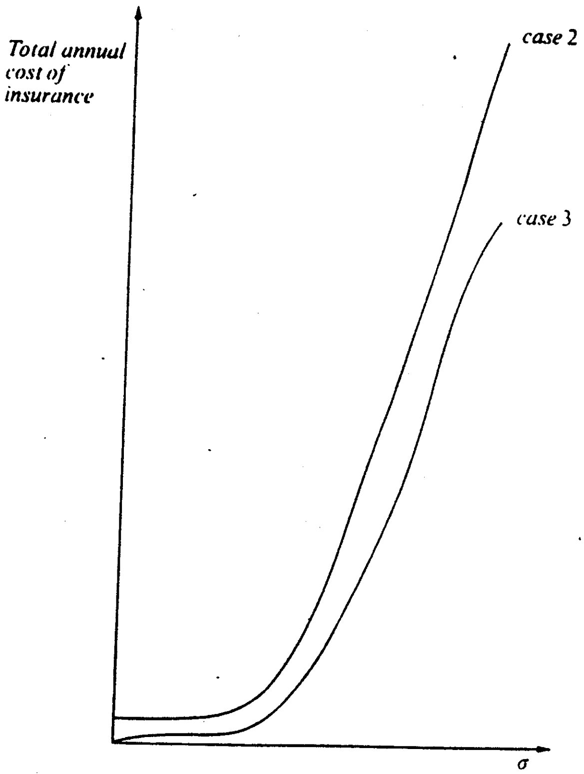

Figure 4. Case 1 has no cost of insurance

These data are shown in Figures 1—4 for the optimal capital budget without insurance (case 1), for the optimal capital budget with required insurance coverage for each item (case 2) and for the optimal combination of risk assumption, risk avoidance and insurance (case 3).

(c2) The Optimal Capital Budget with Different Costs of Insurance

Next we determine the impact of the fixed part of the insurance premium on the optimal capital budget. The fixed insurance premium is varied between DM 200 and DM 4,000 per object and year. The results are given in Figure 5 for different levels of risk.

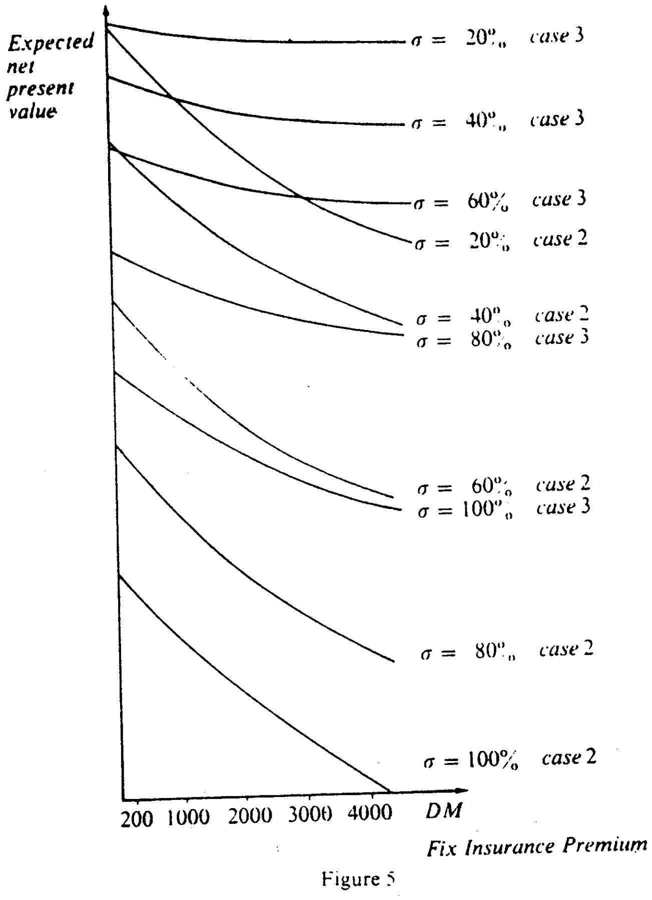

Figure 5

The results given in Figure 5 are plausible. The net present value of the optimal capital budget declines with the cost of insurance going up. However, the decline in the expected net present value with rising fix insurance premium is much more pronounced in case 2 than in case 3. This is due to the fact that the firm can evade rising costs of insurance by investing in projects without covering them by insurance. Secondly it is interesting to note that cases 2 and 3 yield almost identical capital budgets when risk and fix insurance premium are low. Rising risks result in a wide discrepancy between the optimal capital budgets of cases 2 and 3 even when the fix part of the risk premium is low. This is, of course, due to the fact that the variable part of the risk premium rises with increasing risk, and in case 2 there is no other chance of avoiding the rising costs of insurance but not to invest at all.

(c3) The Optimal Capital Budget under Uncertainty and Equity Finance

Equity is in short supply in the examples of the preceding sections.

Figure 6 shows the expected net present value of the optimal capital budget for different levels of equity financing available.

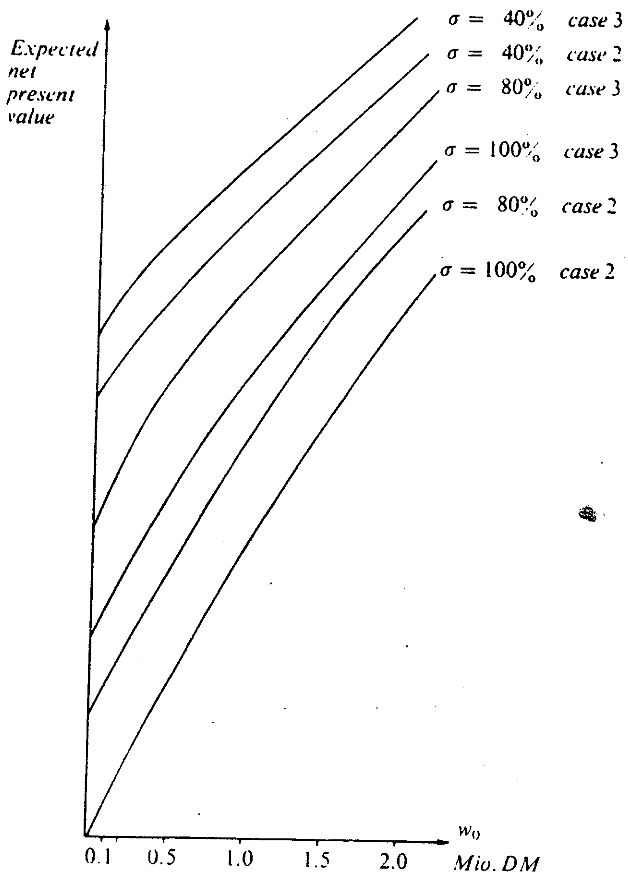

Figure 6

In this section the available funds for equity financing are parametrically changed between 100,000 DM and 2 million deutschmarks.

Figure 6 clearly underlines the fact that equity funds do not only serve as a substitute for expensive credit funds but also for expensive insurance.

It is not intuitively plausible that the marginal expected net present values of the optimal capital budgets should be practically identical for cases 2 and 3. It is the level of the capital budget (and the level of the expected net present value respectively) rather than its rate of increase that is affected by changes in the level of equity money available.

It may be a costly way of substitution for insurance, however, if losses are incurred and have to be borne by the equity funds rather than being spread over a number of companies with similar insurance. Therefore, risk assumption does not become increasingly more desirable in case 3 than in case 2.

(c4) The Optimal Capital Budget with Different Probability Requirements for the Financial Constraints.

The chance-constrained-programming model used here assumes that the firm fixes a probability with which the stochastic constraints are expected to hold. Of course, this subjective probability level is hardly independent of the costs of the risks incurred by the firm. It seems, therefore, at least necessary to study the effect of different probability assignments on the investment outlay and the profitability of the capital budget.

Figure 7 shows the expected net present values of the optimal capital budgets for the three cases under consideration. The risks of the investment proposals σ was assumed as 100% in the computations. It is plausible that the optimal capital budget with obligatory insurance has a lower expected net present value than that budget with a possibility not to cover every object by insurance and that case 1 budgets have the lowest net present values beyond a critical level of risk preference. With rising π insurance becomes more important — and more costly — and with π approaching 100%, investment in risky projects becomes prohibitive — due to the costs involved.

Figure 7 again gives evidence of the importance of the risk management approach to capital budgeting under uncertainty: insurance dominates risk avoidance for every probability level, and risk avoidance, risk assumption and insurance — optimally combined — dominate the other strategies of determining the optimal capital budget under uncertainty.

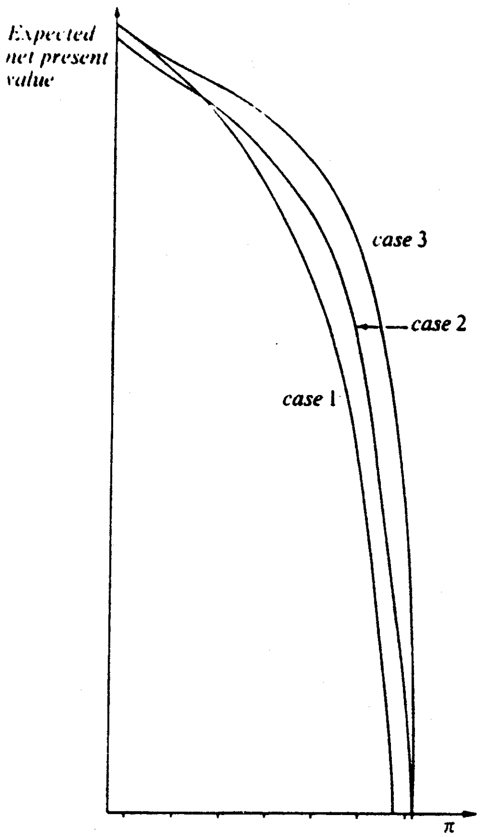

Figure 7

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