Price-Setting and Wage-Setting Firms with Rational Expectations and Adjustment Costs

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CONTENTS

Ⅰ. Introduction

In this paper we retain the idea of firms which set the price and the wage given perceived demand and supply curves, but we now introduce explicit costs of price and wage adsjustment. We also assume that, at the chosen price and wage, firm's expectations of demand and supply are always correct. The nearest to the present model in the existing literature is perhaps Rotemberg(1982), but an important difference is that we allow firms to ration consumers, where they perceive this to be in their inter-

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There is one people to whom I owe an immeasurable debt for helping me to complete this research itself ; President, Choong Sik Chang. I am indebted to Professors J. Driffill at London University and P. Neary at University of Dublin. Any remaining errors are mine.

est.

In model of price-setting and wage-setting firms with adaptive expectations (see K. Shim(1991)), we noted that if expectations were correct, then since the firm would never choose to ration consumers, the economy must be on the boundary between regimes R and K. If we are to assume permanently correct or rational expectations, then consumers can only be rationed of the firm voluntarily chooses to ration them. The only motive which the firm might have for this, it would seem, is that it faces costs of price and wage adjustment. Two types of adjustment cost have commonly been noted: administrative costs, and loss of demand costs. The former are due to the need to direct resources into posting and publicizing new prices; the latter are the less tangible costs due to loss of reputation amongst customers or workers. The cost which we shall assume to exist are most clearly viewed as being of the former type: to incorporate the latter adequately into a general equilibrium model would require a more sophisticated treatment of consumer behavior, and in any case the fear of loss of reputation might advise against deliberate rationing as much as against price changes.

II. Behavior of the Firm

We suppose a single representative firm, with a perceive goods demand and perceived labor supply curve:

$$y_t = P_t^{-\eta} a_t$$
$$l_t = W_t^{\varepsilon} b_t \qquad (1)$$

Note that under rational expectations there is no need te deal explicitly with the case of n firms(see Shim(1991)): for each firm to take account of the prices and wages set by the others is directly overriden if we postulate that his predictions of his own supply and demand are always correct. Price and wage adjustment costs are most simply represented by assuming that changes in the price and the wage enter the firm's production function. With a given level of inputs(i.e. labor), the production function then shows the sacrifice in output which the firm must make if it wishes to change the price and the wage by given amounts. Specifically, we shall assume the production function to have the simple form:

$$y_t = f(l_t) - C, \qquad f' > 0, \quad f'' < 0 \tag{2}$$

where C, the adjustment cost component, is a function of the price and wage changes.

We take C to have the quadratic form:

$$C = \frac{1}{2} c_1 \{\triangle \ln P_{t-1}\}^2 + \frac{1}{2} c_2 \{\triangle \ln W_{t-1}\}^2 \tag{3}$$

where $\triangle$ is the forward difference operator. The features of (3), important for the dynamic behavior of the model, are that adjustment costs are strictly and smoothly convex. This allows the possibility of convergence to a stationary state such as would be attained if there were no adjustment costs. If instead costs were lump-sum or linear, such convergence would not occur.

The firm's decision problem is thus: choose y_t, l_t, P_t, W_t in order to maximize

$$P_t y_t - W_t l_t$$

subject to

$$\begin{aligned} & y_t = f(l_t) - \left[\frac{1}{2} c_1 \{\triangle \ln P_{t-1}\}^2 + \frac{1}{2} c_2 \{\triangle \ln W_{t-1}\}^2\right] && \text{(a)} \\ & y_t \leq P_t^{-\eta} a_t && \text{(b)} \qquad (4) \\ & l_t \leq W_t^{\varepsilon} b_t && \text{(c)} \end{aligned}$$

Note that (4b) and (4c) are now specified as inequalities. Depending on which of them, if any, bind, the firm and thus the economy will be in one of four possible regimes:

Classical unemployment(C)—neither bind

Keynesian unemployment(K)—(4b) binds

Repressed inflation(R)—(4c) binds

Underconsumption(U)—both bind

Let the Lagrangean for this problem be:

$$L = P_t y_t - W_t l_t + \lambda\left[y_t - f(l_t) + \frac{1}{2}c_1\{\triangle \ln P_{t-1}\}^2 + \frac{1}{2}c_2\{\triangle \ln W_{t-1}\}^2\right]$$
$$+ \mu_1\{P_t^{-\eta} a_t - y_t\} + \mu_2\{W_t^{\varepsilon} b_t - l_t\}$$

We then have the first-order conditions:

$$\begin{aligned}
&\partial L/\partial y_t = P_t + \lambda - \mu_1 = 0 && \text{(a)}\\
&\partial L/\partial l_t = -W_t - \lambda f' - \mu_2 = 0 && \text{(b)}\\
&\partial L/\partial P_t = y_t + \lambda c_1\{\triangle \ln P_{t-1}\}/P_t - \mu_1 \eta P_t^{-\mu-1} a_t = 0 && \text{(c)}\\
&\partial L/\partial W_t = -l_t + \lambda c_2\{\triangle \ln W_{t-1}\}/W_t - \mu_2 \varepsilon W_t^{\varepsilon-1} b_t = 0 && \text{(d)}\\
&P_t^{-\eta} a_t - y_t \geq 0, \quad \mu_1\{P_t^{-\eta} a_t - y_t\} = 0 && \text{(e)}\\
&W_t^{\varepsilon} b_t - l_t \geq 0, \quad \mu_2\{W_t^{\varepsilon} b_t - l_t\} = 0 && \text{(f)}
\end{aligned} \qquad (5)$$

To solve these: for each of the complementary slack conditions (5e) and (5f), assume one or other of the inequality holds with strict equality. This gives four possibilities, corresponding to each regime, and we can then solve the implied equations to obtain y_t, l_t, P_t, W_t as implicit functions of P_{t-1}, W_{t-1}, a_t, b_t, for each regime. These equations are summarized below(we omit the production function, (4a), which is common to all):

Regime C : $P_t^{-\eta} a_t > y_t \qquad W_t^{\varepsilon} b_t > l_t$

$$\begin{aligned}
&f'(l_t) = W_t/P_t && \text{(a)}\\
&y_t = c_1 \triangle \ln P_{t-1} && \text{(b)}\\
&l_t f'(l_t) = -c_2 \triangle \ln W_{t-1} && \text{(c)}
\end{aligned} \qquad (6)$$

Regime K : $P_t^{-\eta} a_t = y_t \qquad W_t^{\varepsilon} b_t > l_t$

$$\begin{aligned}
&f'(l_t) = [1 - c_1\{\triangle \ln P_{t-1}\}/\eta y_t]/P_t\{1-1/\eta\} && \text{(a)}\\
&l_t f'(l_t) = -c_2 \triangle \ln W_{t-1} && \text{(b)}
\end{aligned} \qquad (7)$$

$$\text{Regime R}: \quad P_t^{-\eta} a_t > y_t \qquad W_t^{\varepsilon} b_t = l_t$$
$$f'(l_t) = c_2\{\triangle \ln W_{t-1}\}/\varepsilon l_t + \{1+1/\varepsilon\} W_t/P_t \qquad (a)$$
$$y_t = c_1 \triangle \ln P_{t-1} \qquad (b) \qquad (8)$$

$$\text{Regime U}: \quad P_t^{-\eta} a_t = y_t \qquad W_t^{\varepsilon} b_t = l_t$$
$$f'(l_t) = [1 - c_1\{\triangle \ln P_{t-1}\}/\eta y_t] k W_t/P_t + c_2\{\triangle \ln W_{t-1}\}/\varepsilon l_t \qquad (9)$$

where $k \equiv \{1+1/\varepsilon\}/\{1-1/\eta\}$ (see Shim(1991)).

A point to note in (6)−(9) is that, in any regime where the demand constraint (4b) is not binding, the price must be rising(see (6b) and (8b) and likewise in any regime where the supply constraint (4c) is not binding, the wage must be falling (see (6c) and (7b)). This makes intuitive sense, since in the absence of supply or demand constraints, the firm would wish to set $P_t = \infty$ and $W_t = -\infty$ were it not for adjustment costs. Thus, in regime C, P_t is rising and W_t falling, implying W_t/P_t is falling. In K, W_t is falling, while nothing general can be said about P_t. In regime R, P_t is rising, and nothing general can be said about W_t. In regime U, nothing general can be said about either P_t or W_t.

These observations indicate that a stationary state in which neither P_t nor W_t are changing can only occur in regime U. Earlier we noted that with rational expectations but no adjustment costs, the economy must always be at a point where consumers are not rationed, which in the present model also means in regime U. This correspondence of the long-run equilibrium in the adjustment cost model to the temporary equilibrium in the no-adjustment cost model should not surprise us, since the quadratic form of the cost function means that in the quadratic form of the cost function means that in the neighborhood of long-run eqilibrium adjustment costs are zero, i.e. $\partial C/\partial P_t$ and $\partial C/\partial W_t$ are zero.

Let P_t, W_t implicitly defined by (6)−(9) as ;

$$y_t = y(P_{t-1}, W_{t-1}, a_t, b_t) \qquad (a)$$
$$l_t = l(P_{t-1}, W_{t-1}, a_t, b_t) \qquad (b)$$
$$P_t = P(P_{t-1}, W_{t-1}, a_t, b_t) \qquad (c) \qquad (10)$$
$$W_t = W(P_{t-1}, W_{t-1}, a_t, b_t) \qquad (d)$$

Note that these will be kinked at points corresponding to regime boundaries, and that in regimes where (4b) or (4c) hold with strict inequality, the partial derivatives with respect to a_t or b_t(resprctively) wil be zero. However, we shall not directly investigate their comparative static properties, since such an effort is better reserved for the analysis of the complete temporary equlibrium, dealt with below.

III. Behavior of the Consumer

We adopt exactly the same model of the consumer as in model(see, Shim (1991)): the consumer is assumed to act as a passive-taker and quantity-taker in both markets. Since we have assumed a single representative firm, in the consumer's demand and supply functions (11) and (12), "n" should (with no loss of generality in resprenting the behavior of a symmetric n-firm model) be taken to equal unity.

$$\begin{aligned} nc &= c^d(W/P,\ B/P) && \text{(a)} \\ & \quad\ \ (+)\quad (+) && \\ nl &= l^s(W/P,\ B/P) && \text{(b)} \\ & \quad\ \ (?)\quad (-) && \end{aligned} \tag{11}$$

$$\begin{aligned} nc &= \bar{c}^d(B/P + n\bar{l}W/P) && \text{(a)} \\ & \qquad\qquad (+) && \\ nl &= \bar{l}^s(W/P,\ B/P - n\bar{c}) && \text{(b)} \\ & \quad\ \ (?)\quad (+) && \end{aligned} \tag{12}$$

It is useful to introduce the following general notation for the effective demand and supply functions (11) and (12):

$$\bar{\bar{c}}^d(P_t, W_t, \pi_t;\ \bar{l}_t,\ \bar{c}_t) \equiv \begin{cases} c^d(W_t/P_t,\ M_0/P_t + \pi_t - \tau_t) \text{ if } \bar{l}_t \geq \bar{\bar{l}}^s(\cdot) \\ \bar{c}^d(M_0/P_t + \pi_t - \tau_t + l_tW_t/P_t) \text{ if } \bar{l}_t \leq \bar{l}^s(\cdot) \end{cases} \tag{13}$$

$$\bar{\bar{l}}^d(P_t, W_t, \pi_t;\ \bar{l}_t, \bar{c}_t) \equiv \begin{cases} l^d(W_t/P_t, M_0/P_t + \pi_t - \tau_t) \text{ if } \bar{c}_t \geq \bar{\bar{c}}^d(\cdot) \\ \bar{l}^d(W_t/P_t, M_0/P_t + \pi_t - \tau_t - \bar{c}_t) \text{ if } \bar{c}_t \leq \bar{\bar{l}}^d(\cdot) \end{cases}$$

$\bar{\bar{c}}^d(\cdot)$ and $\bar{\bar{l}}^s(\cdot)$ are thus the complete effective demand and supply functions of the consumer when derived according to Benassy's(1975) rule, i.e. by ignoring the quantity constraints on the commodity in question. Like the firm's overall behavioral functions (10), they have kinks at the regime boundaries and in regimes where an quantity constraint is not binding, the partial derivative with respect to this is zero.

Ⅳ. Temporary Equilibrium

The equilibrium concept we shall employ is essentially that of Negish(1961), transferred to a world with price and wage adjustment costs. Given the consumer's and firm's behavioral functions (10) and (13), an equilibrium is a pair (a_t, b_t) such that (we here suppress P_{t-1} and W_{t-1} from the firm's functions):

$$\bar{\bar{c}}^d\{P(a_t, b_t), W(a_t, b_t), \pi_t;\ l(a_t, b_t), y(a_t, b_t)-g_t\}+g_t=P(a_t, b_t)^{-\eta}a_t \qquad (a)$$
$$\bar{\bar{l}}^s\{P(a_t, b_t), W(a_t, b_t), \pi_t;\ l(a_t, b_t), y(a_t, b_t)-g_t\}+g_t=W(a_t, b_t)^{\varepsilon}b_t \qquad (b) \qquad (14)$$

where $\pi_t \equiv y(a_t, b_t)-l(a_t, b_t)W(a_t, b_t)/P(a_t, b_t)$

In other words, to define an equilibrium, we employ the rational expectations assumptions that $\bar{\bar{c}}^d(\cdot)+g_t=(P_t)^{-\eta}a_t$ and $\bar{\bar{l}}^s(\cdot)=(W_t)^{\varepsilon}b_t$

Note that prices and quantities are determined simultaneously in this model. This is an important difference from model of price-setting and wage-setting firms with adaptive expectations(see Shim(1991)), where the price and quantity determination processes were quite distinct. We have therefore introduced a new quantity-constrained equilibrium concept so far employed throughout the thesis. This meets and objection touched on in section II, namely that the traditional concept goes to the opposite extreme of Walrasian equilibrium by making quantity adjustment infinitely fast relative to price adjustment.

To examine the comparative static properties of the economy, we need to look more specifically at the defining equations for each regime. These are summarized below, using the behavioral equations for the firm, (6)−(9), together with the rational expectations assumption for the binding constraints(we again omit the production function, which is common to all):

Regime C

$$f'(l_t) = W_t \quad (a)$$
$$y_t = c_1 \triangle \ln P_{t-1} \quad (b) \qquad (15)$$
$$l_t f'(l_t) = -c_2 \triangle \ln W_{t-1} \quad (c)$$

Regime K

$$y_t = \bar{c}^d(M_0/P_t + y_t + \tau_t) + g_t \quad (a)$$
$$f'(l_t) = [1 - c_1\{\triangle \ln P_{t-1}\}/\eta y_t]/W_t\{1 - 1/\eta\} \quad (b) \qquad (16)$$
$$l_t f'(l_t) = -c_2 \triangle \ln W_{t-1} \quad (c)$$

Regime R

$$l_t = \bar{l}^s(W_t,\ M_0/P_t - W_t l_t - \tau_t + g_t) \quad (a)$$
$$f'(l_t) = c_2\{\triangle \ln W_{t-1}\}/\varepsilon l_t + \{1 + 1/\varepsilon\} W_t \quad (b) \qquad (17)$$
$$y_t = c_1 \triangle \ln P_{t-1} \quad (c)$$

Regime U

$$y_t = c^d(W_t,\ M_0/P_t + y_t - W_t l_t - \tau_t) + g_t \quad (a)$$
$$l_t = l^s(W_t,\ M_0/P_t + y_t - W_t l_t - \tau_t) \quad (b) \qquad (18)$$
$$f'(l_t) = [1 - c_1\{\triangle \ln P_{t-1}\}/\eta y_t] k W_t + c_2\{\triangle \ln W_{t-1}\}/\varepsilon l_t \quad (c)$$

V. Comparative Statics

Consider the impact of a rise in g_t in each of the four regimes. We concentrate on its effect on output and employment, in order to yield comparisons with the Barro-Grossman-Malinvaud model.

In regime C, defined in (13), it is evident that y_t and l_t depend only on P_t, W_{t-1} and not at all on consumer behavior. This is very similar to the fix-price regime C, defined in (19). In both, government spending affects neither output nor employment.

$$
\begin{aligned}
&ny_t = \bar{c}^d(M_0/P_t + ny_t - \tau_t) + g_t,\ l_t = \bar{l}^d(y_t) && (K) \\
&nl_t = \bar{l}^s(W_t, M_0/P_t - W_t l_t - \tau_t + g_t),\ y_t = \bar{y}^s(l_t) && (R) \qquad (19) \\
&y_t = y^s(W_t),\ l_t = l^d(W_t) && (C)
\end{aligned}
$$

In regime K, defined in (16), note that the relationship (16a) carries over from the fix-price model (see 19): output is still demand-determined. However, P_t, which also enters this equation, is now endogenous. Differentiating (16a), we have:

$$
\frac{dy_t}{dg_t} = \frac{1}{1-\bar{c}_y^d} - \frac{m_o \bar{c}_y^d}{1-\bar{c}_y^d}\frac{d\ln P_t}{dg_t} \quad (m_o \equiv M_o/P_t) \qquad (20)
$$

The multiplier is now the sum of two elements: the usual fix-price multiplier, $1/\{1-\bar{c}_y^d\}$, and a price effect. To evaluate the latter, we must totally differentiate the system (16) (together with the production function), when we obtain:

$$
\frac{d\ln P_t}{dg_t} = \frac{-\{lf''+f'\}f' - c_2 f''}{m_o\bar{c}_y^d[-\{lf''+f'\}f' - c_2 f''] - \{1-\bar{c}_y^d\}\{c_1/\eta y + f'\}[\{lf''+f'\}lf' - c_2 f']} \qquad (21)
$$

Here we have evaluated at a point on the stationary locus for P_t; away from this, the derivative also contains terms in $d\ln P_{t-1}$, which necessarily have ambiguous signs. A sufficient condition for this to be positive is that $lf''+f' \leq 0$, i.e. that the elasticity of the marginal product of labor $(-lf''/f')$ be greater than unity. This also ensures that when substituted into (20), the overall multiplier is positive, but less than the simple fix-price multiplier. The case $lf''+f' > 0$ does not immediately falsify these conclusions, but the results are less clear to see. Thus the present model introduces an element of instantaneous inflationary crowding out in the sense that it dampens the fix-price multiplier, but not necessarily in the strict sense that the multiplier is now less than unity.

In regime R, defined in (17), there is a rather more radical change in behavior. From (17c), note that we have:

$$\frac{dy_t}{dg_t} = c_1 \frac{d\ln P_t}{dg_t} \tag{22}$$

Therefore if $d\ln P_t/dg_t > 0$, the intuitively correct result, an iccrease in spending raises output, rather than reduces it as in the fix-price model. To determine the effect on P_t, we totally differentiate (17) (plus the production function), obtaining:

$$\frac{d\ln P_t}{dg_t} = \frac{\tilde{l}^s_y f' \{c_2/\varepsilon l + f'\}}{\{c^2/\varepsilon l + f'\}[\{c_1 + y\}\{1 + \tilde{l}^s_y W\} + f' \tilde{l}^s_y m_o] + \{\tilde{l}^s_w - \tilde{l}^s_y\}[f' c_2/\varepsilon l - f''\{c_1 + y\}]} \tag{23}$$

This is similary evaluated at $\triangle \ln W_{t-1} = 0$. Noting $1 + \tilde{l}^s_y W > 0$ (the counterpart of $1 - \bar{c}^d_y > 0$ and derivable from adding-up plus normality), we have a negative numerator but an ambiguous denominator. Thus a fall in the price, resulting in the usual negative multiplier on output is possible: in either case the outcome is unorthodox.

Since regime U does not exist (except as a limiting case in the form of the R-K boundary) in the Barro-Grossman-Malinvaud model, consider first the properties of the fix-price equilibrium defined by (18a) and (18b) alone. Note that this is determined entirely by the behavioral relations of the consumer — just as regime C is determined entirely by the behavioral relations of the firm. Totally differentiating, we obtain:

$$\frac{dy_t}{dg_t} = \frac{1 + l^s_y W}{1 + l^s_y W - c^d_y} \qquad \text{(a)}$$

$$\frac{dl_t}{dg_t} = \frac{l^s_y}{1 + l^s_y W - c^d_y} \qquad \text{(b)} \tag{24}$$

From adding up and normality, the common denominator of these expressions is positive, while the numerator of the first is positive and of the second negative. Thus an increase in spending has a (multiple) expansionary effect on putput, but a contractionary one on employment. When variations in P_t and W_t are also taken into account, by differentiating the entire system (18) plus the production function we obtain:

$$\frac{dy_t}{dg_t}=f'\frac{l_y^d m_o\{c_2/\varepsilon l+f'\}+W\{l_W^s-l_y^s l\}\{c_2/\varepsilon l-f' c_1/\eta y\}}{[-m_o\{c_y^d-f' l_y^s\}\{c_2/\varepsilon l+f'\}-f''\{c_W^d l_y^s-l_W^s c_y^d\}m_o W-W[\{c_W^d l_y^s-l_W^s c_y^d\}W\{1-k\}+\{c_W^d-c_y^d l\}-f' l_W^s-l_y^s l\}]\{c_2/\varepsilon l-f' c^1/\eta y\}]} \quad (25)$$

evaluating at $\triangle \ln P_{t-1} = \triangle \ln W_{t-1} = 0$. The factor $c_2/\varepsilon l - f' c_1/\eta y$, occurring in the last terms of both numerator and denominator, could be positive or negative. If we ignore it, we obtain $dy_t/dg_t > 0$, which would thus seem the more typical outcome. The employment multiplier is the same, apart from omitting the factor f'. Thus introducing price adjustment preserves the expansionary effect on output, and reverses the contractionary effect on employment.

Ⅵ. Wage and Price Dynamics

We now seek to depict the dynamics of the model on a regime diagram comparable to Figures 1 and 2. Although, in period t, $(M_0/P_t, W_t)$ are no longer exogenous, it is natural to continue to use these as the axes of the regime. Thus we shall view the economy's defining equations (15)−(18) as determining $\triangle \ln P_{t-1}$, $\triangle \ln W_{t-1}$ given P_t, W_t. This means that the phase diagram (see Fig. 3) strictly speaking indicates where the system has come from rather than where it is going to, but this does not invalidate its use as a general guide with the usual provisos about depiciting difference equations.

Figure 3 has been established by making use of what was learnt about how P_t and W_t must be changing, from the discussion of the firm's behavior. There we saw that P_t must be rising in regimes C and R, so it follows that the $\triangle P_t = 0$ locus must lie in the interior of regimes K and U. Similarly we saw that W_t must be falling in re-

Fig 1

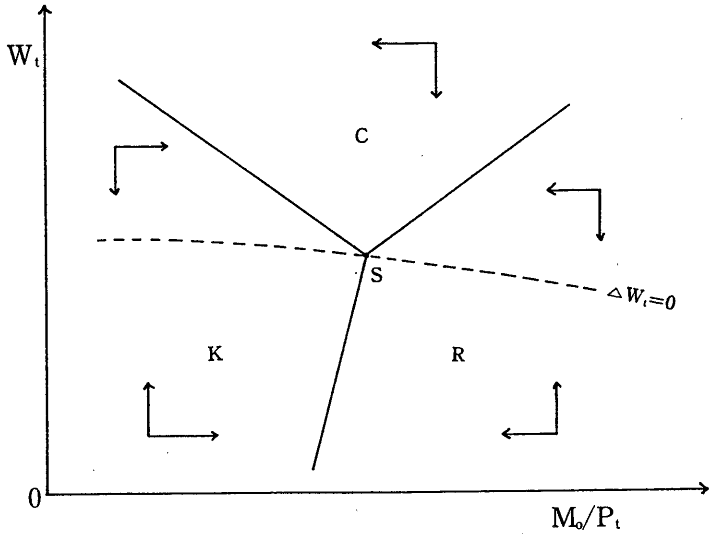
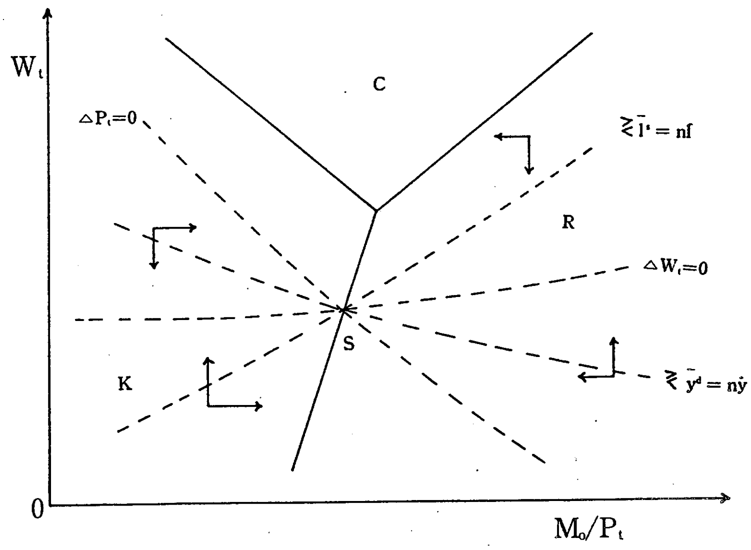


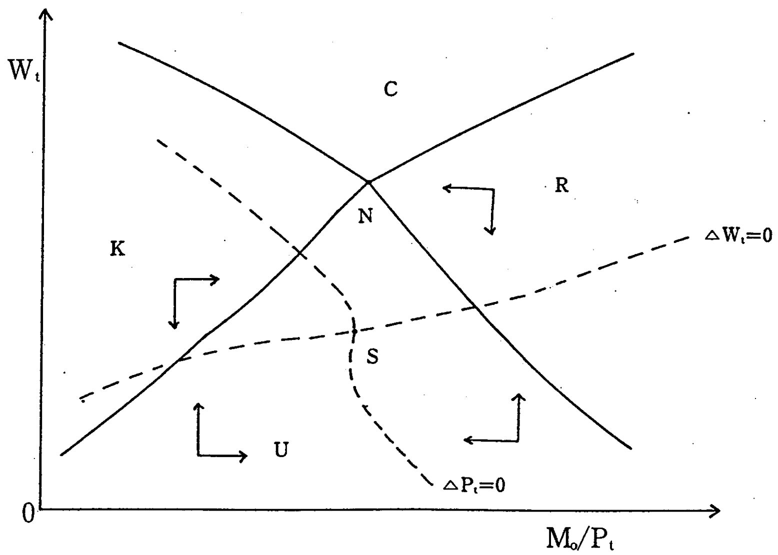
Fig 2



imes C and K, which rules out regime C from containing the $\triangle w_t = 0$ locus. Finally, we saw that al stationary state could only occur in regime U, which must therefore be where the stationary loci intersect. To verify that these stationary points must indeed lie in the interiors and not on the boundaries, we may use the fact that ($\triangle \ln P_{t-1}$, $\triangle \ln W_{t-1}$) are continuous across regime boundaries as may be confirmed by examining each boundary in turn. Thus, for example, S cannot lie on the boundary of regime U, or this point from outside, which would violate continuity. The slopes of the regime boundaries themselves are ambiguous : Fig 3 illustrates a simple case. Note that their common point of intersection or node (N), has no special economic significance, apart from being the point at which both the demand and the supply constraints just bind the firm. In particular, it does not correspond to the usual Walrasian equilibrium. We discuss this further below.

Consider now the comparative statics and local stability of the stationary state S. The equations defining S may be obtained by setting $\triangle \ln P_{t-1} = \triangle \ln W_{t-1} = 0$ in those for regime U, (18). Note that the result is identical to (26), which defines the stationary state in model of price-setting and wage-setting firms with adaptive expectations. That the models are identical in the stationary state is the expected outcome, since the quadratic adjustment cost function ensures both that costs are zero and that their first derivatives are zero in the stationary state. The long-run changes in output and employment produced by a change in spending are exactly as discussed for model of price-setting and wage-setting firms with adaptive expectations(see Shim (1991)) : increases in spending raise the price level and reduce the real wage, and increase both output and employment.

Fig 3



$$
\begin{aligned}
ny^* &= c^d(W^*, M_o/P^* + n\{y^* - w^*l^*\} - \tau) + g && (a)\\
nl^* &= l^s(W^*, M_o/P^* + n\{y^* - w^*l^*\} - \tau) && (b)\\
kW^* &= f'(l^*) && (c)\\
y^* &= f(l^*) && (d)
\end{aligned}
\qquad (26)
$$

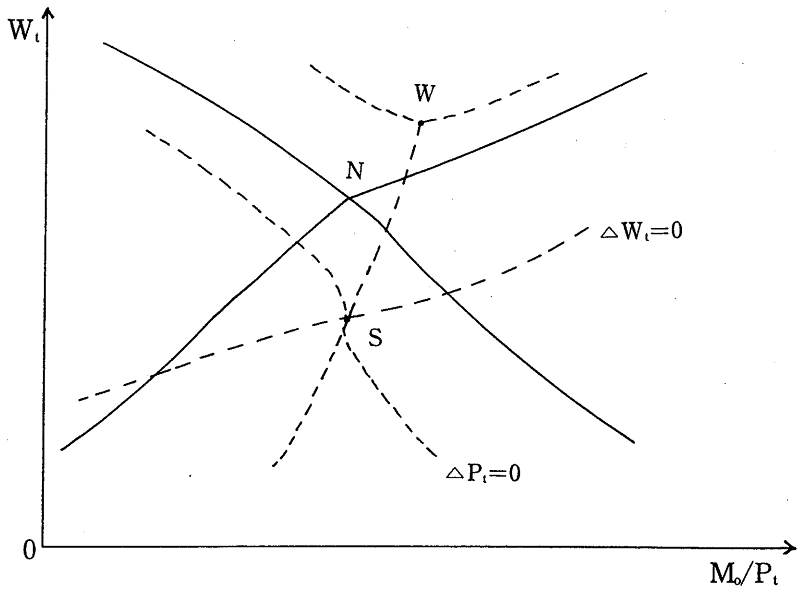
To examine the local stability of S, the usual procedure of linearizing the system (18) about the stationary state, and examining the values of its characteristic roots, is followed. The details are presented in the appendix. It is shown that provided the ambiguous term $c_2/\varepsilon l - f' c_1/\eta y$ (already encountered in the comparative statics of regime U) is negative or not too positive, then satisfaction of the differential equation conditions for stability is guaranteed. The slopes of the stationary loci at S are also derived: that for $\triangle w_t = 0$ is positive, while that for $\triangle P_t = 0$ coincides with the slope of the old R-K boundary on which the point S lies (see Fig 4).

Although we noted above that the common intersection of the regime boundaries does not have the singnificance of being the traditional Walrasian equilibrium, it is of interest too see how it relates to this. The point N is most conveniently defined by observing that it is the only common point of regimes C and U. Thus, combining (18a) and (18b) with (15a) and the production function, we have the equations:

$$
\begin{aligned}
y_t &= c^d(W_t, M_o/P_t + W_t l_t\} - \tau_t) + g_t && (a)\\
l_t &= l^s(W_t, M_o/P_t + W_t l_t\} - \tau_t) && (b)\\
W_t &= f'(l_t) && (c)\\
y_t &= f(l_t) - C && (d)
\end{aligned}
\qquad (27)
$$

These equations are very similar to those defining the Walrasian equilibrium, which we noted ealier were given by (26) whith k set equal to 1. The only difference is the presence of adjustment costs, which we have represented in (18) as the exogenous quantity C although it is of course endogenous, when the full systems (15) and (18) are solved together. By writing it this way and differentiating (27) with respect to C, we may discover how the point N differs in $(M_0/P_t, w_t)$ - space from the Walrasian equilibrium W. Such an exercise(performed in the appendix) shows that P_t must be higher and W_t lower than at W. Thus the relationship of the new regime diagram to the traditional fix-price one is as in Fig. 4 :

Fig 4



Ⅶ. Assessment of the Model and Conclusions

This model is rather farther removed from the tatonnement-augmented one than model of price-setting and wage-setting firms with adaptive expectations. It shares the same stationary state as model of price-setting and wage-setting firms with adaptive expectations which is not vastly dissimilar to the Walrasian stationary state in the tatonnement model. However, the wage and price adjustment process has now substantially modified the temporary equilibrium of the original fix-price model, which was carried over intact in model of price-setting and wage-setting firms with adaptive expectations (see Shim(1991)). In particular, we now have simultaneous price and quantity determination, and also a fourth rationing regime: the underconsumption regime, as labelled by Muellbauer and Portes(1978).

In certain respects, model of price-setting and wage-setting firms with rational expectations and adjustment costs has significant advantages over model of price-setting and wage-setting firms with adaptive expectations. Any involuntary unemployment is no longer due to incorrect expectations, and in particular, the adjustment process is not dependent on the particular rule of thumb by which expectations are formed. Note also that we no longer even need to appeal to Iwai's kind of argument to justify the assumption that the wage and price are inflexible : adjustment costs provide an explicit justification for this. Another improvement is that the model makes price and quantity adjustment simultaneous, rather than being split into an artificial sequance as in model of price-setting and wage-setting firms with adaptive expectations(see Shim(1991)). Finally, when extended to incorporate asset dynamics, the model provides a framework in which the costs of inflation can be explicitly determined—an important omission in most model of inflation.

From the policy point of view, the most important feature of the model presented in this paper, as of the tatonnement-augmented model which they in many ways resemble, is that they both result in the economy converging over time to a unique full employment state. That the common long-run equilibrium is essentially a full employment state should not need much arguing. The consumer is not rationed in either goods or labor markets at such a point, while the firm is only rationed in the voluntary sense that it is deliberately restricting its labor demand and goods supply in order to take advantage of perceived monopoly/monopsony power. The long-run equilibrium is in fact a monopolistically competitive equilibrium of the standard Negishi (1961) type. Thus the long-run effects of increases on g or cuts in τ—with the important qualification that we are ignoring the dynamics of the government deficit--are more or less identical to those in Walrasian equilibrium, which is to say expansionary but limited by the capacity to slide the economy up its labor supply curve. Moreover if we imagine the simpler case of an exogenous labor supply then these effects are zero(see Shim(1991)). In such a case the role of fiscal policy, at least as regards output and employment, is reduced to one of assisting the adjustment process to a new long-run equilibrium following an exogenous shock.

Appendix

(1) stability of the stationary state

In regime U, write y_t and l_t as the following functions,

$$\tilde{y}^U(\ln P_t, \ln w_t),\ l_t = \tilde{y}^U(\ln P_t, \ln w_t) \qquad (28)$$

which are implicitly defined by (18a) and (18b). These functions were examined earlier for their comparative static properties with respect to g_t; the same exercise enables us to evaluate their partial derivatives with respect to $\ln P_t$ and $\ln w_t$ as:

$$\tilde{y}^U_{W.} = -c^d_y m_o/\{1+l^s_y W-c^d_y\} < 0$$

$$\tilde{l}^U_P = -l^s_y m_o/\{1+l^s_y W-c^d_y\} > 0$$

$$\tilde{y}^U_W = w[\,w\{c^d_W l^s_y - l^d_W - c^d_y\}] + \{c^d_W l_y - c^d_y l\}/\{1+l^s_y w-c^d_y\} = ? \qquad (29)$$

$$\tilde{l}^U_W = w[\,w\{c^d_W l^s_y - l^d_W - c^d_y\}] + \{l^s_W l_y - l^s_y l\}/\{1+l^s_y w-c^d_y\} = ?$$

The defining equations (18) for regime U plus the production function can then be written more succinctly as:

$$\tilde{y}^U(\ln P_t, \ln w_t) = f[\,l^U(\ln P_t, \ln w_t)] - [\,1/2c_1\{\triangle \ln P_{t-1}\}^2 + 1/2c_2\{\triangle \ln w_{t-1}\}^2] \qquad (30)$$

$$f'[\,l^U(\ln P_t, \ln w_t] = [\,1 - c_1\{\triangle \ln P_{t-1}\}/\eta\, y^U(\ln P_t, \ln w_t)]\,kw_t$$
$$+\ c_2\{\triangle \ln W_{t-1}\}/\varepsilon \tilde{l}^U(\ln P_t, \ln w_t) \qquad (31)$$

This gives us two implicit first-order difference equations in $\ln P_t$, $\ln w_t$. To satisfy the differential equation conditions for local stability, we need that the matrix of partial derivatives whose general form is an in (32), evaluated at the stationary state, S, should have eigenvalues both of whose real parts are negative. The matrix must thus have a positive determinant and a negative trace.

$$\begin{bmatrix} (\partial \triangle \ln P_t / \partial \ln W_t) & (\partial \triangle \ln P_t / \partial \ln W_t) \\ (\partial \triangle \ln P_t / \partial \ln W_t) & (\partial \triangle \ln P_t / \partial \ln W_t) \end{bmatrix} \quad (32)$$

To compute the matrix, first totally differentiate the above equations tresting $\triangle \ln P_{t-1}$, $\triangle \ln W_{t-1}$ as endogenous and $\ln P_t$, $\ln w_t$ as exogenous, and then rearrange terms to obtain :

$$\begin{bmatrix} \hat{y}_P^U - f' I_P^U & \hat{y}_W^U - f' I_W^U \\ c_2/\varepsilon l - f' c_1/\eta y - f'' I_P^U & c_2/\varepsilon l + f' - f'' I_W^U \end{bmatrix} \begin{bmatrix} \triangle \ln P_{t-1} \\ \triangle \ln W_{t-1} \end{bmatrix} = \begin{bmatrix} -\hat{y}_P^U + f' I_P^U & -\hat{y}_W^U + f' I_W^U \\ f'' I_P^U & -f' + f'' I_W^U \end{bmatrix} \begin{bmatrix} d\ln P_{t-1} \\ d\ln W_{t-1} \end{bmatrix} \quad (33)$$

Let us write this in matrix notation as :

$$\underline{A}\, d\triangle \underline{x}_{t-1} = \underline{B}\, d\triangle \underline{x}_{t-1}$$

or

$$d\triangle \underline{x}_{t-1} = \underline{A}^{-1} \underline{B}\, d\underline{x}_{t-1}$$

For stability we need $\det(\underline{A}^{-1}\underline{B} > 0$, $\mathrm{tr}(\underline{A}^{-1}\underline{B}) < 0$, Now, $\det(\underline{A}^{-1}\underline{B}) = \det \underline{B} / \det \underline{A}$.

We have :

$$\det \underline{B} = \underset{(-)}{f'} \{\underset{(-)}{\hat{y}_P^U - f' I_P^U}\} + \underset{(-)}{f''} \{\underset{(+)}{I_P^U \hat{y}_W^U - I_W^U \hat{y}_P^U}\}$$

$$\det \underline{A} = \det \underline{B} + \underset{(-)}{\{\hat{y}_P^U - f' I_P^U\}}\, c_2/\varepsilon l - \underset{(-)}{\{\hat{y}_W^U - f' I_W^U\}}\, \{c_2/\varepsilon l - f'\, c_1/\eta y\}$$

Using the definitions for I_P^U, etc. listed above, it is straightforward to show that the second right hand side term $\{\cdot\}$ in $\det \underline{B}$ is positive, whence $\det \underline{B}$ is unambiguously negative. Given this, $\det \underline{A}$ is also negative if we ignore the third term on the right hand side. This term contains the fundamentally ambiguous factor $c_2/\varepsilon l - f' c_1/\eta y$ and the factor $\hat{y}^U_W - f' \hat{l}^U_W$ whose sign is most likely to be negative, as we see below. Thus if $c_2/\varepsilon l - f' c_1/\eta y$ is negative or not too positive, the condition $\det(\underline{A}^{-1}\underline{B} > 0$ is satisfied.

Noting that $\mathrm{tr}(\underline{A}^{-1}\underline{B}) = \partial \triangle \ln P_{t-1} / \partial \ln P_{t-1} + \partial \triangle \ln W_{t-1} / \partial \ln W_{t-1}$, these terms may be computed from (33) using Cramer's rule, and we obtain quite simply:

$$\mathrm{tr}\,(\underline{A}^{-1}\underline{B}) = -1 - \det \underline{B} / \det \underline{A}$$

The condition $\mathrm{tr}(\underline{A}^{-1}\underline{B}) < 0$ is therefore satisfied if $\det(\underline{A}^{-1}\underline{B}) > 0$ is satisfied.

To examine the slopes of the stationary loci in $(\ln P_t, \ln w_t)$-space, set $\triangle \ln P_{t-1} = \triangle \ln W_{t-1} = 0$ in (33), thus obtaining:

$$\left.\frac{d\ln w_t}{d\ln P_t}\right|_{\triangle P = 0} = -\frac{\hat{y}^U_P - f' \hat{l}^U_P}{\hat{y}^U_W - f' \hat{l}^U_W} \qquad \text{(a)}$$

$$\text{(34)}$$

$$\left.\frac{d\ln w_t}{d\ln P_t}\right|_{\triangle W = 0} = -\frac{f'' \hat{l}^U_P}{f' - f'' \hat{l}^U_W} < 0 \qquad \text{(b)}$$

Equation (34a) is simply the slope of old R-K boundary definable as $\hat{y}^U(\cdot) = f(\hat{l}^U(\cdot))$, which we saw was positive(negative, in (P_t, w_t)—space at the Walrasian equilibrium. At S, the same slope does not necessarily hold, but for S sufficiently close to W it would do so, as drawn in Fig. 4 and in this case, $\hat{y}^U_W - f' \hat{l}^U_W < 0$.

(2) Position of the node

Using the notation of the previous section, the equation (29) defining the node can be re-written as:

$$\begin{aligned} W &= f' \ell^U(P,\ w) \\ \hat{y}^U(P,\ w) &= f(\ell^U(P,\ w)) - C \end{aligned} \qquad (35)$$

using levels rather than logs for convenience. Totally differentiating,

$$\frac{dw}{dC} = \frac{\overset{(-)}{f''\ell^U_P}}{-\{y_P - f'\ \ell^U_P\} + f''\{\ell^U_W \hat{y}^U_P - \ell^U_P \hat{y}^U_W} < 0 \qquad (a)$$

$$\frac{dP}{dC} = \frac{f''\ell^U_P}{-f''\ell^U_P\{\hat{y}^U_W - f'\ \ell^U_W\} - \{1 - f''\ell^U_W\}\{\hat{y}^U_P - f'\ \ell^U_P} \qquad (b)$$

(36)

The sign of (36a) is readily determined as shown, using $\ell^U_W \hat{y}^U_P - \ell^U_P \hat{y}^U_W < 0$. In (36a), $1 - f''\ell^U_P$ has an ambiguous sign. However, note that the denominator of (36a) is just a simplification of that of (36b), which therefore is also positive. Now, assuming the R-K boundary is positively sloped in (m_0, w-space, i. e. that (34a) is negative, then from (34a) we have $\hat{y}^U_P - f'\ \ell^U_P$, $\hat{y}^U_W - f'\ \ell^U_W < 0$. Together with a positive denominator this implies that $1 - f''\ell^U_W > 0$ and thus that (34b) is positive.

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