Asset Prices with Beliefs, Doubts and Learning

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Benchmark : Standard C-CAPM

- income shock process : $y_{t+1} \sim P\left(\cdot \mid y_t\right),\ y_t \equiv \left(\kappa_t, w_t\right)$
- equilibrium asset price :

$$p_t = E_t\left[m_{t+1}\left(p_{t+1} + \kappa_{t+1}\right)\right]$$

  stochastic discount factor : $m_{t+1} = \beta u'\left(C_{t+1}\right) / u'\left(C_t\right)$

- first order condition for representative investor's optimal consumption-savings choice - Euler equation :

$$p_t u'\left(c_t\right) = \beta E_t\left[u'\left(c_{t+1}\right)\left(p_{t+1} + \kappa_{t+1}\right)\right]$$

  periodic budget equation : $c_t = c\left(a_{t+1}, a_t, y_t, p_t\right)$

- How could investors purchase an asset taking as given its expected rate of return, which depends on their purchases ?

Rational Expectations Equilibrium

- Each investor chooses an optimal decision rule under his belief about the joint probability distribution of unrealized variables conditioned on the realized states.
  - Investors know all the realized states - no hidden state.
  - investor's perception, or model, of the way the stochastic law of motion for aggregate variables is determined :

    $$A_{t+1} = D\left(X_t\right),\ p_t = p\left(X_t\right),\ y_{t+1} \sim P\left(\cdot \mid y_t\right)$$

    where $X_t \equiv (A_t, y_t)$ is the vector of aggregate states.
  - recursive optimal control - Bellman equation :

    $$V\left(a_t, X_t\right) \equiv \max_{a_{t+1}} \left\{ u\left(c\left(a_{t+1}, a_t, y_t, p\left(X_t\right)\right)\right) + \beta \int V\left(a_{t+1}, X_{t+1}\right) F\left(dX_{t+1} \mid X_t\right) \right\}$$

  - individual decision rule : $a_{t+1} = d\left(a_t, X_t\right)$

Rational Expectations Equilibrium

- Rational expectations impose that the actual probabilities of unrealized variables should coincide with investors' beliefs.
  - Investors know the law of motion for shocks.
  - actual aggregate decision rule : $A_{t+1}^{*} = D^{*}(X_t) \equiv d(A_t, X_t)$
  - mutual consistency between investors' choices and what their perceptions are of aggregate choices : $D = D^{*} = T(D)$
  - market clearing with $D^{*} = D \Rightarrow p^{*} = p$
- Each investor's belief is an equilibrium outcome.
  - Bayesian Nash equilibrium under incomplete information
- Investors should trust their model because it is the true model. - equivalent to the case that they know the true model.
  - "Big K, little k trick" for representative agent models

Expected Utility Hypothesis

- Under rational expectations, subjective and objective probabilities coincide.
- lottery : random variable $\tilde{x} : S \mapsto X$ defined on (S, Σ, P)
  affine utility function : $u : X \mapsto \mathbb{R}$
- aggregating utility values over states : $U(\tilde{x}) = \mathcal{R}(u \circ \tilde{x})$
- von Neumann-Morgenstern expected utility preferences

$$\mathcal{R}(\xi) = \int \xi P(d\xi) \Leftrightarrow U(\tilde{x}) = E[u(\tilde{x})]$$

  - risk attitudes
  - discounted expected utility : timing indifference

Asset Pricing Anomalies

- Equity Premium Puzzle and Risk-Free Rate Puzzle
- Equity volatility is too high to be justified by changes in the fundamental.
- Excess returns are serially correlated, mean reverting, and forecastable.

- Price-dividend ratios move procyclically, and conditional expected equity premiums move countercyclically.
- Conditional volatility of stock returns is persistent and moves countercyclically.

- It looks like that investors attach more weight on low continuation values in recessions. - pessimism

Alternative Hypothesis of Choice under Risk

- Allais paradox
- descriptive theories vs. non-expected utility theories
- framing of outcomes and reference dependence
    - loss aversion - prospect theory
    - aspiration criterion - SP/A theory
- sensitivity toward probability
    - probability weighting
    - rank-dependent utility
    - Choquet expected utility
    - probability transformation in the cumulative prospect theory
    - decumulatively weighted utility in the SP/A theory

Knightian Uncertainty and Ambiguity Attitudes

- choice under unknown probabilities - insufficient data
- subjective expected utility preferences
- Ellsberg paradox
- ambiguity and ambiguity attitudes
  - non-additive prior : Choquet expected utility
  - multiple prior
- plausibility - evidence theory
  - belief function
  - fuzzification
- discernment - fuzzy decision theory
  - membership function and fuzzy measure
  - fuzzy rule-based choice

Timing Attitudes

- four aspects of dynamic stochastic model
  - intertemporal elasticity of substitution - MRS btwn periods
  - risk aversion - MRS btwn states
  - preference for earlier resolution of uncertainty
  - aversion to long-run risk
- recursive utility : Kreps-Porteus, Epstein-Zin
- preferences
  - discounted expected utility : $V_t = u(c_t) + \beta E[V_{t+1}]$
  - recursive utility : $V_t = W(u(c_t), E[V_{t+1}])$
  - discounted ambiguity aversion utility : $V_t = u(c_t) + \beta \mathcal{R}(V_{t+1})$
  - generalized ambiguity aversion utility : $V_t = W(u(c_t), \mathcal{R}(V_{t+1}))$
- interdependence of ambiguity and timing

Doubts about Beliefs

- information and degree of ambiguity
  - Bayesian theory : conditional distribution on hidden states
  - plausibility
  - discernment
- model misspecification under imperfect information
  - in the underlying stochastic law for the hidden states
  - of probabilities assigned to the hidden states
- Could agents in a model be endowed with more precise information than econometricians ?
- heuristics vs. choice under ambiguity
- robust control with constraint or multiplier preferences
  - realtive entropy, i.e. Kullback-Leibler divergence : information loss when a model is used to describe reality

Learning - Updating Ambiguous Beliefs

- Bayes' rule for the SEU preferences
- Bayesian approach - filtering in a hidden Markov model
  - compound lottery in which the probability of each outcome is the expected value under given prior over hidden states
  - time-varying Markov states in a regime-switching model
  - A recursive implementation of Bayes' rule gives a new Markov process with a distribution over the hidden states. - no fears of model misspecification
  - ambiguity aversion : robust estimation and filtering
- Dempster-Shafer updating
  - maximum likelihood updating
  - prior-by-prior updating
- updating fuzzy rule by learning