Learning about Risk and Return under Diverse Beliefs

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Motivation

Bubbles & Crashes

- Branch and Evans (AEJ 2011) demonstrates that an asset pricing model with least squares learning can lead to bubbles and crashes as endogenous responses to the fundamentals.
- A central role is played by changes over time in agents' *estimates of risk* referred to as perceived risk.

Why do *rational agents* care about bubbles?

- high-order beliefs in Keynes beauty contests.

Dose belief diversity produce *excess volatility*?

- What kind of belief diversity has nonvanishing aggregate effects?
- rational diverse beliefs & market volatility

Adaptive Learning

Adaptation of Beliefs in a *Self-Referential System*

- dynamic predictor selection - evolutionary & probabilistic
- adaptive learning

Self-Confirming Equilibrium & Recursive Learning Algorithm

- Fudenberg and Levine (EC 1993, JET 2009), Sargent (AER 2008)
- *bounded rationality* in the sense of Sargent (1993)

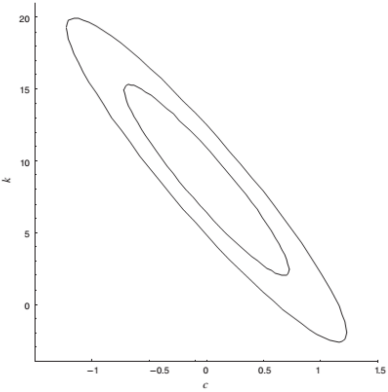
Real-Time Econometric Learning

- parametric or nonparametric estimation of a forecasting model
- feedback between *perceived law of motion* & *actual law of motion*
- cognitive consistency principle - Evans and Honkapohja (2013)

Persistent Learning Dynamics

Because costant-gain learning weights recent data more heavily, convergence is to a random fixed point.

Escape Dynamics in Branch and Evans (AEJ 2011): $p_t = k + cp_{t-1} + \varepsilon_t$



Rational Belief

Heterogenous Beliefs about an *Exogenous Process*

- asymmetric information
- rational diversity

Why do rational agents have wrong beliefs?

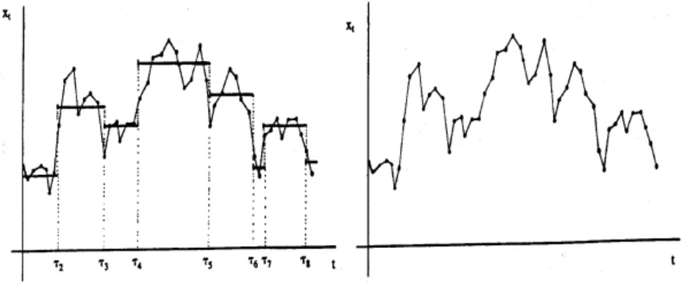
A belief is rational if it is *compatible* with the empirical evidence.

Example

- A black box contains two coins.
- Toss a coin randomly drawn from the box.
- *empirical probability* of heads: 0.50
- Belief $(1/3)(0.60) + (2/3)(0.45)$ is rational.
  - *frequency* of optimism: 1/3, *intensity* of optimism: 0.60
- rational overconfidence

Statistical Stability

Nonstationary Regime-Switching Process - Kurz (1997)



- short investment horizon & long-term statistics

If a stochastic process is statistically stable and ergodic, its asymptotic empirical distribution behaves as if it were induced by a stationary ergodic process.

Simple Linear Asset Pricing Model

continuum of investors, discrete time indexed by the integer

two assets: a risky asset (stock) & a risk free asset

- publicly traded in Walrasian auctions
- stock: ex-dividend price p_t, stochastic dividend d_t per share
- risk free asset: fixed gross rate of return R

beliefs or forecasts

- conditional expectation & variance E_t, V_t
- investor type h's beliefs E_{ht}, V_{ht}, $h \in \{1, 2, \cdots, H\}$
- investor type h' s proportion $n_h \in (0, 1)$

supply of shares per investor $\{z_t\}$

Simple Linear Asset Pricing Model

At time t, the demand per investor of type h for shares is given by

$$z_{ht} = \left(\alpha\sigma_{ht}^2\right)^{-1}\left(E_{ht}\left[p_{t+1} + d_{t+1}\right] - Rp_t\right),$$

where $\sigma_{ht}^2 \equiv V_{ht}\left[p_{t+1} + d_{t+1}\right]$. $\alpha\sigma_{ht}^2$ indicates the *perceived risk*.

The market-clearing price of the stock is given by

$$Rp_t = \sum_{h=1}^{H} n_h \left(\frac{\sigma_{ht}^2}{\sigma_t^2}\right)^{-1} E_{ht}\left[p_{t+1} + d_{t+1}\right] - \alpha\sigma_t^2 z_t,$$

where $\alpha\sigma_t^2$ is the harmonic mean of all investors' perceived risks.

Exogenous Processes

fundamental of the stock y_t

The dividend stream is driven by

$$\begin{aligned} d_t &= \beta y_t + \mu_u + u_t, \\ y_t &= \rho y_{t-1} + \mu_\varepsilon + \epsilon_t, \end{aligned}$$

where u_t, ϵ_t are uncorrelated i.i.d. normal disturbances with means zero and variances σ_u^2 and σ_ϵ^2.

The share supply follows

$$z_t = \min\left\{1, \frac{p_t}{\varsigma p^\star}\right\} \cdot \left(\mu_v + v_t\right),$$

where $0 < \varsigma < 1$, $p^\star$ is the *long-run fundamental price*, and v_t is an i.i.d. disturbance with mean zero and variance σ_v^2, uncorrelated with u_t, ϵ_t.

Belief Diversity

At time t, every investor observes the history of publicly available information $\{p_s, d_s, y_s; s \leq t\}$.

Investors' PLM for the fundamental of the stock is

$$y_t = \rho y_{t-1} + \mu_\epsilon + \delta_t + \hat{\epsilon}_t, \tag{1}$$

where $\hat{\epsilon}_t$ is an identically distributed zero-mean disturbance, and δ_t represents a perturbation to the mean of $\mu_\epsilon + \hat{\epsilon}_t$.

At time t, investor type h forms a belief or personal opinion that the deviation will be

$$x_{ht} \equiv E_{ht}\left[\delta_{t+1}\right] = E_{ht}\left[y_{t+1}\right] - \rho y_t - E\left[y_{t+1} - \rho y_t\right].$$

If $x_{ht} > 0$, type h investors are *optimistic* at time t in the sense that they are expecting unusually high dividend in the next period.

Rational Beliefs

For every type h, the dynamics of its state of belief x_{ht} is specified by

$$x_{ht} = \lambda x_{ht-1} + \xi_{ht} \tag{2}$$

where ξ_{ht} is an i.i.d. normal disturbance with mean zero and variance σ_{ξ}^2, uncorrelated with $\hat{\epsilon}_t$.

Definition

All the investor's beliefs are rational if with probability one the PLM (1) with $\delta_t = x_{ht-1}$ and the stochastic dynamics of x_{ht} in (2) generates a realization of $\{y_t\}$ in which every empirical distribution of a finite dimension is the same in the actual realization.

The dynamics of state of market belief $X_t \equiv \sum_h n_h x_{ht}$ is given by

$$X_t = \lambda X_{t-1} + \xi_t,$$

where $\xi_t \equiv \sum_h n_h \xi_{ht}$. Let ξ_t be i.i.d. with mean zero and variance σ_{ξ}^2.

Perceived Return

At time t, investor type h's forecast on next period's dividend is

$$E_{ht}d_{t+1} = \beta\rho y_t + \beta\mu_\epsilon + \mu_u + x_{ht}.$$

Investors' PLM for stock price is in the form of a state-space model

$$\begin{aligned} p_t &= \phi y_t + \psi X_t + \mu_\varepsilon + \varepsilon_t \\ X_t &= \delta X_{t-1} + \hat{\xi}_t, \end{aligned}$$

where ε_t, $\hat{\xi}_t$ are white-noise disturbances, or of a MSV solution

$$p_t = \delta p_{t-1} + \phi\left(y_t - \delta y_{t-1}\right) + \left(1-\delta\right)\mu_\varepsilon + \psi\hat{\xi}_t + \varepsilon_t - \delta\varepsilon_{t-1}. \qquad (3)$$

At time t, investors' forecast on next period's stock price is

$$E_{ht}p_{t+1} = \delta p_t + \phi\left(\rho - \delta\right)y_t + \left(1-\delta\right)\mu_\varepsilon + \phi\mu_\epsilon.$$

Rational Belief Equilibrium

With the forecasts on the first moments, investors' perceive risk

$$\sigma_{ht}^2 = E_{ht}\left(p_{t+1} - E_{ht}p_{t+1} + d_{t+1} - E_{ht}d_{t+1}\right)^2$$

does not depend on a type, and is time-invariant.

The market clearing condition implies that

$$(R-\delta)\,p_t = [\phi\,(\rho-\delta) + \beta\rho]\,y_t + \mu + \psi\,(1-\lambda L)^{-1}\,\xi_t - \alpha\sigma^2 v_t$$

where $\mu \equiv (1-\delta)\,\mu_\varepsilon + (\phi+\beta)\,\mu_\epsilon + \mu_u + \alpha\sigma^2\mu_v$.

The PLM (3) with $\varepsilon_t = \alpha\sigma^2 v_t,\ \hat{\xi}_t = \xi_t$,

$$\phi = (R-\delta)^{-1}\,\beta\rho,\ \ \mu_\varepsilon = (R-\delta)^{-1}\left[(\phi+\beta)\,\mu_\epsilon + \mu_u\right],\ \ \delta = \lambda$$

constitutes a RBE.