May Monetary Policy Affect to Long Run Expectation of Non-Stationary Real Interest Rate ?

by

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Abstract

In this paper, we first introduce an augmented non-stationarity test of real interest rate within a cointegrated VAR (vector autoregressive) model of interest rate, inflation and output gap reflecting the New Keynesian frame work. Suggesting test additionally checks whether the cointegration coefficients of inflation and output gap are 1 and 0 over the conventional cointegration test. We show an interest rate shock trend including monetary policy shock (INTTREND) may be extracted from a non-stationary real interest rate using Beveridge-Nelson decomposition. We suggest a test to check the existence of INTTREND in the real interest rate and show that a long run effect of monetary policy shock to the real interest rate may be estimated consistently. According to the empirical analyses using monthly US data, we can reject the null that there is not an INTTREND in the non-stationary real interest rate with 1% significance level. We can observe that (i) 1% increase of federal fund target rate may approximately induce 0.4% increase of the real interest rate's long run expectation (RELEX), (ii) relatively higher RELEX than the federal fund rate or ex post real interest rate just after global financial crisis (2007) that may explain why the US economy has not rapidly recovered.

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1. Introduction

After the global financial crisis, major central banks of the United States, European Union and Japan are conducting monetary policies to sustain continuously low interest rates to boost the economy. However note not the nominal but the real interest rate is crucial for the economy including investment and asset price determination. It is because the interest rate is a determinant of the level of consumption and investment, while any economic agent with the rational expectation will conduct her/his decision making based on not the nominal interest rate but the expected inflation deducted real interest rate. Obviously, a monetary policy may affect to the nominal interest rate in general. However such potential of monetary policy to the real interest rate is not obvious because the real interest rate is composed of the expected inflation as well as the nominal interest rate.

On the causal relation between the real interest rate and monetary policy, Shiller (1979) examined three hypotheses[2] on the ineffectiveness of the monetary policy on the real interest rate under the rational expectation hypothesis. He concluded that none of the hypothesis is likely to be so strictly correct as to rule out completely a predictable effect of systematic monetary policy on expected real interest rates.

The effectiveness of the monetary policy on the real interest rate is somewhat related with the statistical property of real interest rate. Fama(1975) regarded the real interest rate as a constant and from there analyzed the efficiency of the bond market. If the real interest rate is a constant, then the monetary policy can not change the real interest rate. However, according to Garbade and Watchel(1978) and Nelson and Schwert(1977), the real interest rate is not a constant but an integrated I(1) process. Rose(1988) concluded that an ex-ante real interest rate is an I(1) process with a unit root in the Fisher equation because the nominal interest rate is I(1) and the inflation is I(0). Walsh(1987) also regarded the ex-ante real interest rate as a unit root process.

The inflation rate is often empirically observed to be an I(1) process. In this case, if the nominal interest rate and inflation are co-integrated, so then the real interest rate is regarded as I(0). Examples of the studies that have tested whether the real interest rate is stationary through the cointegration approach are MacDonald and Murphy(1989), Mishkin(1992), Wallace and Warner(1993), Crowder and Hoffman(1996), Koustas and Serletis(1999) and Rapach and Wohar(2004), among others.

According to Walsh(1987), if an ex ante real interest rate has a unit root, then it has an important implication as an index of monetary policy. For instance, any sustaining trend change in real interest rates might require a

[2] *Hypothesis* 1: The Fed has no ability to shock rationally expected real interest rates at all in the short run or long run (c.f., Fama, 1975). *Hypothesis* 2: The Fed can shock rationally expected real interest rates, but only by taking policy actions other than the actions the public supposes they are taking (c.f., Lucas, 1973; Sargent and Wallace, 1975). *Hypothesis* 3: This hypothesis 3 depends on the length of the policy effectiveness interval. If the interval is years long, then the Fed may have substantial scope for systematic countercyclical monetary policy (c.f., Phelps and Taylor,1977; Fischer, 1977).

more general policy reaction rather than a temporary adjustment. Under this circumstance, a long run monetary policy effect to the real interest rate becomes important.

In this paper, we first introduce an augmented non-stationarity test of real interest rate within a cointegrated VAR model of interest rate, inflation and output gap reflecting the New Keynesian frame work. Suggesting test additionally checks whether the cointegration coefficients of inflation and output gap are 1 and 0 over the conventional cointegration test. We show an interest rate shock trend including monetary policy shock (INTTREND) may be extracted from a non-stationary real interest rate using Beveridge-Nelson decomposition. We suggest a test to check the existence of INTTREND in the real interest rate and show that a long run effect of monetary policy shock to the real interest rate may be estimated consistently.

The rest of this paper proceeds as follows. Section 2 discusses the decomposition of real interest rate considering a New Keynesian framework. Section 3 focuses on the extraction of stochastic trends from non-stationary real interest rate. Section 4 introduces the inference for stochastic trends in real interest rate. The empirical analysis result for the US data is discussed in Section 5. Section 6 contains the conclusion.

2. Cointegrated VAR model of inflation, output gap and interest rate

2.1 New Keynesian frame work with a dynamics of interest rate

In this section, we extract a non-stationary part from a real interest rate where the interest rate and inflation are determined from a New Keynesian frame work. Cochrane (2016, p9) introduced a standard optimizing sticky price model as

(2.1) $g_{t-1} = E_{t-1}g_t - \sigma(i_{t-1} - E_{t-1}\pi_t)$

(2.2) $\pi_{t-1} = \beta E_{t-1}\pi_t - \kappa g_{t-1}$,

where E_{t-1} denotes a conditional expectation at time t-1, i_t is the nominal interest rate at time t, inflation π_t is at time t made at time t-1,[3] and g_t denotes the output gap at time t.[4] Note that (2.1) is an inter-temporal substitution condition which may be generalized by adding the time derivative of the interest rate in the continuous time expression [c.f., Cochrane; 2016 equation (8)]. Note that (2.2) denotes Phillips curve. By definition, we may decompose

[3] The interest rate i_t and inflation π_t are defined for a deposit and price change for the time from t to t+1. So i_t is fixed at time t while π_t is a random variable at time t-1.

[4] Cochrane(2016) calculated the impulse response of inflation and the output gap to a step rise in the interest rate using (2.1) and (2.2) and find inflation rises through out the episode.

(2.3) $g_t = E_{t-1}g_t + \delta_t^g$

and

(2.4) $\pi_t = E_{t-1}\pi_t + \delta_t^\pi$

where δ_t^g is a unexpected shock to output gap with $E_{t-1}\delta_t^g = 0$ and δ_t^π is a unexpected shock to the inflation with $E_{t-1}\delta_t^\pi = 0$. If we plug these definitions of conditional expectations of (2.3) and (2.4) into (2.1) and (2.2), then we get following dynamic equations

(2.5) $\sigma\pi_t + g_t = g_{t-1} + \sigma i_{t-1} + \sigma\delta_t^\pi + \delta_t^g$

(2.6) $\beta\pi_t = \pi_{t-1} + \kappa g_{t-1} + \beta\delta_t^\pi$,

after some arrangements.

We then suppose that the interest rate is determined by following equation;[5]

(2.7) $i_t = -\phi_1 g_t - \phi_2\pi_t + \psi_1 g_{t-1} + \psi_2\pi_{t-1} + \psi_3 i_{t-1} + \omega_1\delta_t^\pi + \omega_2\delta_t^g + \mu_t$

where μ_t is a unexpected shock to the interest rate with $E_{t-1}\mu_t = 0$. For instance, μ_t includes a monetary policy shock and risk premium. Note (2.7) is so general to cover all variables contained in (2.5) and (2.6). If $\phi_2 = \phi_2 = \psi_1 = \psi_2 = \psi_3 = \omega_1 = \omega_2 = 0$, then $i_t = \mu_t$ from (2.7) and the interest rate is determined by purely exogenous monetary policy shock as in Cochrane (2016, p9).

If we collect these equations (2.5)-(2.7), then following VAR model is given;

(2.8) $$\begin{pmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ \phi_1 & \phi_2 & 1 \end{pmatrix} Z_t = \begin{pmatrix} 1 & \kappa & 0 \\ 0 & 1 & \sigma \\ \psi_1 & \psi_2 & \psi_3 \end{pmatrix} Z_{t-1} + \begin{pmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ \omega_1 & \omega_2 & 1 \end{pmatrix} \zeta_t .$$

where $Z_t \equiv (\pi_t, g_t, i_t)'$ and $\zeta_t = (\delta_t^\pi, \delta_t^g, \mu_t)'$.

Now we suppose

Assumption 2.1 $(\zeta_t)_{t=1}^n$ *is an independent, identically and normally distributed sequence with a distribution as*;

$$\zeta_t \sim N\left[0, \begin{pmatrix} \Sigma_\pi & 0 & 0 \\ 0 & \Sigma_g & 0 \\ 0 & 0 & \Sigma_\mu \end{pmatrix}\right].$$

Then note (2.8) may be rewritten as a following structural VAR(1) model

[5] See Mishikin (2012) for the determination of interest rate at the money market equilibrium.

$$
(2.9)\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \phi_1-\omega_1 & \phi_2-\omega_2 & 1 \end{pmatrix} Z_t = \begin{pmatrix} \frac{1}{\beta} & \frac{\kappa}{\beta} & 0 \\ -\frac{\sigma}{\beta} & 1-\frac{\sigma\kappa}{\beta} & \sigma \\ \frac{\sigma\omega_2-\omega_1+\beta\psi_1}{\beta} & \frac{(\sigma\omega_2-\omega_1)\kappa+\beta(\psi_2-\omega_2)}{\beta} & \sigma(1-\omega_2) \end{pmatrix} Z_{t-1} + \zeta_t
$$

using

$$
\begin{pmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ \omega_1 & \omega_2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\beta} & 0 & 0 \\ -\frac{\sigma}{\beta} & 1 & 0 \\ \sigma\omega_2-\omega_1 & -\omega_2 & 1 \end{pmatrix}.
$$

The identification order implied by the recursive system (2.9), a la Sims (1980), is from the inflation, output gap and the nominal interest rate. This causality ordering has following interpretations. First, the nominal interest rate (mostly determined by the monetary policy) may not contemporaneously affect to the output gap and inflation considering standard price rigidity and policy effect lag. However the output gap and inflation may contemporaneously affect to the (nominal) interest rate through the instant change of future expected short term interest rate. Note, according to the expectation theory of interest rate, a long term interest rate is determined by the average of future expected short term interest rates.[6] The inflation and output gap are not contemporaneously correlated with each other under above New Keynesian frame work.

Finally, (2.9) may be written as a reduced form VAR(1) model as;

$$
(2.10)\quad Z_t = \Pi_1 Z_{t-1} + \varepsilon_t
$$

where

$$
\Pi_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \phi_1-\omega_1 & \phi_2-\omega_2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{\beta} & \frac{\kappa}{\beta} & 0 \\ -\frac{\sigma}{\beta} & 1-\frac{\sigma\kappa}{\beta} & \sigma \\ \frac{\sigma\omega_2-\omega_1+\beta\psi_1}{\beta} & \frac{(\sigma\omega_2-\omega_1)\kappa+\beta(\psi_2-\omega_2)}{\beta} & \sigma(1-\omega_2) \end{pmatrix}
$$

and

[6] Note the change of expectation may immediately change the interest rate. For instance, just after D. Trump's surprise win in the US presidential election, investors have begun to question their long-held consensus forecasts for subdued inflation and mediocre growth that underpinned a rally in bonds. Yield on US government debt has risen from record lows and fixed income investors have lost huge amount since the election.

(2.11) $\xi_t = (\omega_1 - \phi_1)\zeta_t^{\pi} + (\omega_2 - \phi_2)\zeta_t^{g} + \mu_t = \delta_t' \begin{pmatrix} \omega_1 - \phi_1 \\ \omega_2 - \phi_2 \end{pmatrix} + \mu_t$

and $\varepsilon_t = (\delta_t', \xi_t)'$ with $\delta_t = (\zeta_t^{\pi}, \zeta_t^{g})'$ because

$$\varepsilon_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \phi_1 - \omega_1 & \phi_2 - \omega_2 & 1 \end{pmatrix}^{-1} \zeta_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \omega_1 - \phi_1 & \omega_2 - \phi_2 & \omega_3 \end{pmatrix} \zeta_t .$$

In this paper, we assume that Z_t (including the inflation and interest rate) is I(1). Then structural VAR(1) model (2.10) may be generalized as a reduced form (possibly cointegrated) VAR model(q) as follows:

(2.12) $Z_t = \Pi_0 + \sum_{i=1}^{q} \Pi_i Z_{t-i} + \varepsilon_t$ or $\Delta Z_t = \Pi_0 + \Phi Z_{t-1} + \sum_{i=1}^{q-1} \Phi_i \Delta_i Z_{t-i} + \varepsilon_t$,

where Π_i is a 3×3 coefficient matrix for $i = 1,2,\cdots,q$, $\Delta Z_t = Z_t - Z_{t-1}$, $\overline{\Pi} = \sum_{i=1}^{q} \Pi_i$, $\Phi = \overline{\Pi} - I_3$ and $\Phi_i = -(\Pi_{i+1} + \Pi_{i+2} + \cdots + \Pi_q)$ where the co-integration is represented by the singularity restriction (Johansen, 1991) of the long-run impact matrix Φ, as follows:

Assumption 2.2 *Suppose that* $\Phi = \alpha\beta$, *where* $\underset{1\times 2}{\beta = (-\gamma', 1)}$ *is a cointegration vector with* $\gamma \neq 0$.[7]

2.2 A sufficient condition for non-stationarity of real interest rate

As the definition of the real interest rate, we introduce the Fisher equation:

(2.13) $i_t = r_t + \pi_t^e$

where r_t is the real interest rate at time t and π_t^e is the expectation of inflation at time t. Following Rose (1986) and Rapach and Weber (2004), we assume that

Assumption 2.3 *The inflation expectation error* $\pi_t^e - \pi_t$ *is* I(0).

[7] The case when the number of cointegration vector is two may be similarly analyzed. In this paper, we do not restrict the cointegration vector β.

Assumption 2.3 may be justified under the rational expectation hypothesis of Lucas-Sargent-Wallace, which states that there is not a systematic error in the inflation expectation. According to Campbell and Shiller (1988a, b), if y_2 is I(1) and y_1 is a rational forecast of future value of y_2, then y_1 and y_2 will be cointegrated. Under this logic, if π_t is I(1) and π_t^e is a rational forecast of future value of π_t, then π_t and π_t^e are cointegrated where conformable cointegration error $\pi_t^e - \pi_t$ is I(0).

Proposition 2.4 *Suppose* Assumption 2.3 *holds. Then real interest rate* r_t *is* I(1) *if* $\gamma^{*}{}'x_t$ *is* I(1) *where* $\gamma^* \equiv \gamma - (1,0)' \equiv (\gamma_1, \gamma_2)' - (1,0)'$ and $x_t \equiv (\pi_t, g_t)'$.[8]

Proof: Note we may rewrite Fisher equation (2.2) as

(2.14) $r_t = i_t - \gamma' x_t + \gamma^{*}{}' x_t - (\pi_t^e - \pi_t)$.

Note $i_t - \gamma' x_t$ is I(0) because $(1, -\gamma')'$ is a co-integration vector and $\pi_t^e - \pi_t$ is I(0) under Assumption 2.3. So the real interest rate r_t can be I(1) if $\gamma^{*}{}' x_t$ is I(1). Q.E.D.

So $\gamma^{*}{}' x_t$ is a unique I(1) part of r_t in (2.14) and thus $\gamma^* = 0$ is a sufficient condition for a non-stationary real interest rate from (2.14). So our approach has two different aspects even after a cointegration test (without unrestricted cointegration vector) for Z_t. First, we will explicitly test $\gamma_1 = 1$. This hypothesis is not conducted conventionally. For instance, see Rapach and Wohar (case iii in p414, a cointegration test in p423; 2004). However, as we see in decomposition (2.14), a cointegration in Z_t is not sufficient for the stationarity of real interest rate even if $\gamma_2 = 0$. Second, we admit the real interest rate might be affected by the output gap a la New Keynesian frame work. So we additionally test the null hypothesis $\gamma_2 = 0$.

In following section, we decompose a non-stationary part of real interest rate $\gamma^{*}{}' x_t$ to deduce the stochastic trends therein.

[8] When an interest tax of exogenous tax rate τ is considered, then the co-integration vector might be modified [e.g., Neely and Rapach(2008)].

3. Extraction of stochastic trends from non-stationary real interest rate

To extract I(1) trends from a real interest rate, we consider a transformed VAR model of (2.12):

(3.1) $TZ_t = T\Pi_0 + \sum_{i=1}^{q}(T\Pi_i T^{-1})TZ_{t-i} + e_t$,

with a transformation matrix with a co-integration vector as:

(3.2) $T \equiv \begin{pmatrix} I_2 & 0 \\ -\gamma' & 1 \end{pmatrix}$, $|T| \neq 0$,

where $u_t = i_t - \gamma' x_t$, $TZ_t = (x_t', u_t)'$ and $e_t = T\varepsilon_t = (\delta_t, \xi_{\gamma t})'$, with $\xi_{\gamma t} \equiv \xi_t - \gamma\delta_t$.

Then, following Kim (2012), we may transform the model (3.1) into a VAR model of the stationary variable under Assumption 2.2 as follows:

(3.3) $\begin{pmatrix} \Delta x_t \\ u_t \end{pmatrix} = \Psi_0 + \sum_{i=1}^{q} \Psi_i \begin{pmatrix} \Delta x_{t-i} \\ u_{t-i} \end{pmatrix} + \begin{pmatrix} \delta_t \\ \xi_{\gamma t} \end{pmatrix}$,

with $\Psi_{11q} = 0$ and $\Psi_{21q} = 0$ where

$\Psi_0 \equiv T\Pi_0$ and $\Psi_i \equiv \begin{pmatrix} \underset{2\times 2}{\Psi_{11i}} & \underset{2\times 1}{\Psi_{12i}} \\ \underset{1\times 2}{\Psi_{21i}} & \underset{1\times 1}{\Psi_{22i}} \end{pmatrix}$ for i =1,2,...,q.

Then, if invertible, Model (3.3) may also be written as a vector moving average form:

(3.4) $\begin{pmatrix} \Delta x_t \\ u_t \end{pmatrix} = \begin{pmatrix} \psi_{01} \\ \psi_{02} \end{pmatrix} + \begin{pmatrix} \theta_{11}(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{pmatrix} \begin{pmatrix} \delta_t \\ \beta' e_t \end{pmatrix}$,

where L is a time lag index and

$[I_3 - \sum_{i=1}^{q} \Psi_i]^{-1} \Psi_0 = \begin{pmatrix} \psi_{01} \\ \psi_{02} \end{pmatrix}$ and

$$\begin{pmatrix} \theta_{11}(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{pmatrix} \equiv \begin{pmatrix} I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i & -\sum_{i=1}^{q} \Psi_{12i} L^i \\ -\sum_{i=1}^{q-1} \Psi_{21i} L^i & 1 - \sum_{i=1}^{q} \Psi_{22i} L^i \end{pmatrix}^{-1},$$

where

$$\begin{aligned} \theta_{11}(L) &\equiv [(I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i + \sum_{i=1}^{q-1} \Psi_{12i} L^i)(1 - \sum_{i=1}^{q} \Psi_{22i} L^i)^{-1} (\sum_{i=1}^{q-1} \Psi_{21i} L^i)]^{-1} \\ &\equiv \sum_{i=0}^{\infty} L^i \theta_{11i} \end{aligned}$$

and

$$\theta_{12}(L) \equiv (I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i)^{-1} (\sum_{i=1}^{q} \Psi_{12i} L^i) \times$$
$$[1 - \sum_{i=1}^{q} \Psi_{22i} L^i - (\sum_{i=1}^{q-1} \Psi_{21i} L^i)(I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i)^{-1} (\sum_{i=1}^{q} \Psi_{12i} L^i)]^{-1}$$
$$\equiv \sum_{i=0}^{\infty} L^i \theta_{12i}.$$

Then, from (3.4), a Beveridge-Nelson (BN) decomposition of x_t is defined as[9]

$$(3.5) \quad x_t = x_0 + \sum_{s=1}^{t} \Delta x_s = x_0 + \theta_{11}(1) \sum_{s=1}^{t} \delta_s + \theta_{12}(1) \sum_{s=1}^{t} \xi_{\gamma s} + \Psi_{01} t + \eta_t - \eta_0 ,$$

where $\Delta x_t = \theta_{11}(L)\delta_t + \theta_{12}(L)\xi_{\gamma t}$ and $\sum_{t=1}^{\infty} t^{1/2} |\theta_{1it}| < \infty$ for i=1,2 and; η_t is a stationary process. Then we finally get the BN decomposition of non-stationary part of real interest rate as[10]

$$(3.6) \quad \gamma^{*}{}' x_t = \gamma^{*}{}' x_0 + \gamma^{*}{}' \theta_{11}(1) \sum_{s=1}^{t} \delta_s + \gamma^{*}{}' \theta_{12}(1) \sum_{s=1}^{t} \xi_{\gamma s} + \gamma^{*}{}' \Psi_{01} t + \gamma^{*}{}' (\eta_t - \eta_0)$$

after pre-multiplying $\gamma^{*}{}'$ on the left of equation (3.5).

Now to extract the interest rate shock trend that is orthogonal with the output gap and inflation trends as,

$$(3.7) \quad \gamma^{*}{}' x_t = \gamma^{*}{}' x_0 + \gamma^{*}{}' [\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}'] \sum_{s=1}^{t} \delta_s + \gamma^{*}{}' \theta_{12}(1) \sum_{s=1}^{t} \tilde{\xi}_s + \gamma^{*}{}' \Psi_{01} t + \gamma^{*}{}' (\eta_t - \eta_0)$$

where $\bar{\gamma} \equiv (E\delta_t \delta_t')^{-1} E\delta_t \xi_{\gamma t}$ and $\tilde{\xi}_t \equiv \xi_{\gamma t} - \bar{\gamma}' \delta_t$. Note $\bar{\gamma}$ is a non-zero population projection coefficient that satisfies $E\tilde{\xi}\delta_t = 0$. Thus the shock $\tilde{\xi}_t$, which is independent of the shock δ_t under the normality of ε_t . Then we show the shock $\tilde{\xi}_t$ is equivalent with the structural shock to the interest rate in (2.7).

Lemma 3.1 *Suppose* Assumption 2.1 *holds*. *Then* $\tilde{\xi}_t = \mu_t$.

Proof: Note

$$(3.8) \quad \tilde{\xi}_t \equiv \xi_{\gamma t} - \bar{\gamma}' \delta_t = \xi_t - \gamma' \delta_t - (\lambda - \gamma)' \delta_t = \xi_t - \lambda' \delta_t$$

because $\bar{\gamma} \equiv (E\delta_t \delta_t')^{-1} E\delta_t \xi_{\gamma t} = (E\delta_t \delta_t')^{-1} E\delta_t (\xi_t - \delta_t' \gamma) = \lambda - \gamma$ where

$$(3.9) \quad \lambda \equiv (E\delta_t \delta_t')^{-1} E\delta_t \xi_t = (E\delta_t \delta_t')^{-1} E\delta_t \left[\delta_t' \begin{pmatrix} \kappa_1 - \phi_1 \\ \kappa_2 - \phi_2 \end{pmatrix} + \mu_t \right] = \begin{pmatrix} \kappa_1 - \phi_1 \\ \kappa_2 - \phi_2 \end{pmatrix}.$$

from (2.11) for the second equality and $E\delta_t \mu_t = 0$ from Assumption 2.1 for the third equality. Therefore we get the claimed result because

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[9] For reference, see Hamilton (1994, *pp*545-546).

[10] Watson (1986) suggested that the long-horizon conditional forecast used to compute the BN trend corresponds to an estimate of the permanent component of an integrated series.

$$\tilde{\xi}_t = \xi_t - \lambda' \delta_t = \xi_t - \begin{pmatrix} \kappa_1 - \phi_1 \\ \kappa_2 - \phi_2 \end{pmatrix}' \delta_t = \mu_t$$

from (2.11), (3.8) and (3.9). Q.E.D.

Now, from Lemma 3.1, we may rewrite (3.7) as

(3.10) $\gamma^*' x_t = \gamma^*' x_0 + \gamma^*'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}'] \sum_{s=1}^{t} \delta_s + \gamma^*' \theta_{12}(1) \sum_{s=1}^{t} \mu_s + \gamma^*' \psi_{01} t + \gamma^*'(\eta_t - \eta_0)$

where $\gamma^*' \theta_{12}(1) \sum_{s=1}^{t} \mu_s$ is an interest rate shock trend (INTTREND) and $\gamma^*'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}'] \sum_{s=1}^{t} \delta_t$ is the trend of inflation and output gap shocks (INFTREND, OUTPUTTREND). Note the stochastic trend of real interest rate may be also interpreted as a long run conditional expectation of real interest rate as;

(3.11) $\lim_{j \to \infty} E_t\left[r_{t+j} - \gamma^*' \psi_{01}(t+j) \right] = \underbrace{\gamma^*'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}'] \sum_{s=1}^{t} \delta_s}_{\text{INFTREND+OUTPUTTREN D}} + \underbrace{\gamma^*' \theta_{12}(1) \sum_{s=1}^{t} \mu_s}_{\text{INTTREND}}$

assuming $\eta_0 = 0$ because $\gamma^*' \eta_t$ is I(0).

From (3.11), the long run responses of conditional expectation for different shocks are given as;

(3.12) $\dfrac{\partial \lim_{j \to \infty} E_t r_{t+j}}{\partial \mu_t} = \gamma^*' \theta_{12}(1)$

for the interest rate (including monetary policy) shock and

(3.13) $\dfrac{\partial \lim_{j \to \infty} E_t r_{t+j}}{\partial \delta_t} = \gamma^*'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']$

for the output gap and inflation shocks, from (3.11).

In following section, we will discuss on the estimation of the above trends and to test the existence of INTTREND in a real interest rate.

4. Inference for stochastic trends

To estimate the above trends, we take the following steps (c.f., Kim; 2014, 2016):

1. Estimate Π_i as $\hat{\Pi}_i$ for all $i = 1, 2, \cdots, q$ from the VAR model (3.1) and get the residual $(\hat{\delta}_t, \hat{\xi}_t)'$; $t = 1, 2, \cdots, n.$

2. Estimate the co-integration coefficient β as $\hat{\beta} = (-\hat{\gamma}', 1)$ by Johansen (1991) and estimate γ^* as $\hat{\gamma}^* \equiv \hat{\gamma} - (0,1)'$.

3. Estimate λ, $\bar{\gamma}$ and $\tilde{\xi}_t$ as $\hat{\lambda} \equiv (\sum_{s=1}^{n} \hat{\delta}_s \hat{\delta}_s')^{-1} \sum_{s=1}^{n} \hat{\delta}_s \hat{\xi}_s$, $\hat{\bar{\gamma}} = \hat{\lambda} - \hat{\gamma}$ and $\hat{\tilde{\xi}}_t = \hat{\xi}_t - \hat{\bar{\gamma}}' \hat{\delta}_t$ from (3.8).

4. Estimate the co-integration error u_t as $\hat{u}_t = \hat{\beta}' z_t$.

5. Run an OLS regression (3.3) replacing u_t into $\hat{u}_t$ to get the estimators of Ψ_i as $\hat{\Psi}_i$ for $i = 0,1,2,\cdots,q$.

6. Compute the estimator of $\theta_{11}(1)$ and $\theta_{12}(1)$ in (3.5) as

$$\hat{\theta}_{11}(1) = [I_2 - \sum_{i=1}^{q-1}\hat{\Psi}_{11i} + (\sum_{i=1}^{q-1}\hat{\Psi}_{12i})(1 - \sum_{i=1}^{q}\hat{\Psi}_{22i})^{-1}(\sum_{i=1}^{q-1}\hat{\Psi}_{21i})]^{-1};$$

$$\hat{\theta}_{12}(1) = (I_2 - \sum_{i=1}^{q-1}\hat{\Psi}_{11i})^{-1}(\sum_{i=1}^{q}\hat{\Psi}_{12i}) \times$$
$$[I_2 - \sum_{i=1}^{q}\hat{\Psi}_{22i} - (\sum_{i=1}^{q-1}\hat{\Psi}_{21i})(I_2 - \sum_{i=1}^{q-1}\hat{\Psi}_{11i})^{-1}(\sum_{i=1}^{q}\hat{\Psi}_{12i})]^{-1}.$$

7. Estimate the trends at time *t* as:

$$\hat{\gamma}^{*}'[\hat{\theta}_{11}(1) + \hat{\theta}_{12}(1)\hat{\bar{\gamma}}']\sum_{s=1}^{t}\hat{\delta}_s \quad \text{and} \quad \hat{\gamma}^{*}'\hat{\theta}_{12}(1)\sum_{s=1}^{t}\hat{\tilde{\xi}}_s .$$

Now we suggest testing the existence of INTTREND in the real interest rate. Note the INTTREND does not exist when $\gamma^{*} = 0$ or $\theta_{12}(1) = 0$ from (3.11). First, to test the null hypothesis $\gamma^{*} = 0$ or $\gamma = (0,1)'$, we may exploit log-liklihood ratio test in Johansen (1991).

Second, note the null hypothesis $\theta_{12}(1) = 0$ holds if and only if

(4.1) $\sum_{i=1}^{q}\underset{2\times1}{\Psi_{12i}} = 0$,

from (3.4) where $(I_2 - \sum_{i=1}^{q-1}\Psi_{11i})^{-1}$ and $[1 - \sum_{i=1}^{q}\Psi_{22i} - (\sum_{i=1}^{q-1}\Psi_{21i})(I_2 - \sum_{i=1}^{q-1}\Psi_{11i})^{-1}(\sum_{i=1}^{q}\Psi_{12i})]^{-1}$ are not singular. It is noteworthy that the equality (4.1) holds if the co-integration disequilibrium error u_t does not block Granger cause to the fundamental change Δx_t (or $\Psi_{12i} = 0$ for any *i*=1,2,...,*q*).

To test the null hypothesis in (4.1), we rewrite the equations for Δx_t in (3.3) as:

(4.2) $$\Delta x_t = \Psi_0 + \sum_{i=1}^{q-1}\Psi_{11i}\Delta x_{t-i} + \sum_{i=1}^{q}\Psi_{12i}u_{t-i} + \delta_t,$$
$$= \Psi_0 + \sum_{i=1}^{q-1}\Psi_{11i}\Delta x_{t-i} + \Gamma u_{t-1} + \sum_{i=1}^{q-1}\Gamma_i \Delta u_{t-i} + \delta_t$$

where $\Gamma = \sum_{i=1}^{q}\Psi_{12i}$ and $\Gamma_i = -\sum_{j=1}^{q}\Psi_{12i+j}$. Therefore, a test of the null hypothesis $H_0 : \sum_{i=1}^{q}\Psi_{12i} = 0$ in (4.1) is equivalent to that $H_0 : \Gamma = 0$ in (4.2).

To construct a test statistic for this null hypothesis, define $B = (\underset{2\times1}{\Psi_0}\ \underset{2\times2}{\Psi_{111}},...,\underset{2\times2}{\Psi_{12q-1}},\underset{2\times1}{\Gamma},\underset{2\times1}{\Gamma_1},...,\underset{2\times1}{\Gamma_{q-1}})$, which is the component coefficient matrices in (4.1). Further, define a stacked variables $\Delta x_{-i} = (\Delta x_{t-i+1}, \Delta x_{t-i-1}, \cdots, \Delta x_{t-i-T})'$, $u_{-i} = (u_{t-i+1}, u_{t-i-1}, \cdots, u_{t-i-T})'$ and

$Z = (\mathrm{i}, \Delta x_{-1}, \Delta x_{-2}, ..., , \Delta x_{q-1}, \hat{u}_{-1}, \Delta\hat{u}_{-2}, \cdots, \Delta\hat{u}_{q-1})$ with $\underset{n\times 1}{\mathrm{i}} \equiv (1,1,...,1)$. The OLS estimator of B' becomes $B' = (Z'Z)^{-1}Z'\Delta x_0$. Finally, the Wald test for $H_0 : \Gamma = 0$ may be written as (Lütkepohl , 1993, p93)

$$h_2 \equiv n(C \times vec(\hat{B}))'[C(\hat{\Xi}^{-1} \otimes \hat{\Sigma})C']^{-1} C \times vec(\hat{B}),$$

where n is a smaple number, $\hat{\Xi} = Z'Z / n$, $\hat{\Sigma} = (\Delta x_0 - \hat{B}Z')'(\Delta x_0 - \hat{B}Z')/n$ and $C = \begin{pmatrix} \underset{2\times 4(q-1)+1}{0} & I_2 & \underset{2\times 2(q-1)}{0} \end{pmatrix}$ is a selection matrix of Γ in $vec(B)$.

Theorem 4.1 *Suppose (i)* $H_0 : \Gamma = 0$ *hold, (ii)* $p\lim\hat{\Xi}$ and $p\lim\hat{\Sigma}$ *are both non-singular (iii)* $\Phi = \alpha\beta'$. *Then*

(a) $h_2 \xrightarrow{d} \chi_2^2$.

(b) $\hat{\gamma}^{*}{}'[\hat{\theta}_{11}(1) + \hat{\theta}_{12}(1)\hat{\bar{\gamma}}']\sum_{s=1}^{t}\hat{\delta}_s \xrightarrow{p} \gamma^{*}{}'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s$

and $\hat{\gamma}^{*}{}'\hat{\theta}_{12}(1)\sum_{s=1}^{t}\hat{\tilde{\xi}}_s \xrightarrow{p} \gamma^{*}{}'\theta_{12}(1)\sum_{s=1}^{t}\tilde{\xi}_s$ *for any given t.*

See Kim (2014, 2016; Theorem 3) for the proof. Theorem 4.1 holds mainly due to the super-consistency of the co-integration coefficient $\hat{\gamma}$.

5. Empirical application for the United States data

In this section, we conduct inference for the trends in real interest rate using monthly data of the United States. The data source is FRED of Federal Reserve Bank of St. Louis. The nominal interest rate is 1-year treasury constant maturity rate (percent, not seasonally adjusted) and industrial production index (index 2012=100, seasonally adjusted) is log transformed. The inflation is computed by the 100 times difference of the log consumer price index and its 1-year lagged one where consumer price index is for all urban consumers (all items, index 1982-1984=100, seasonally adjusted). The output gap is defined as the difference between actual and potential industrial production index[11] where potential industrial production is estimated using a linear trend.[12] See Brouwer (1998) for the issues on the output gap estimation. The data period is from 1953.4 to 2016.12 because the interest rate is just available after 1953.4 in FRED.

[11] The GDP is often used for the computation of output gap while it is not available by a monthly frequency.
[12] The other Hodrick-Prescott filter method to estimate an output gap has the disadvantage that the selection of the smoothing weight is arbitrary. So we did not used this filter.

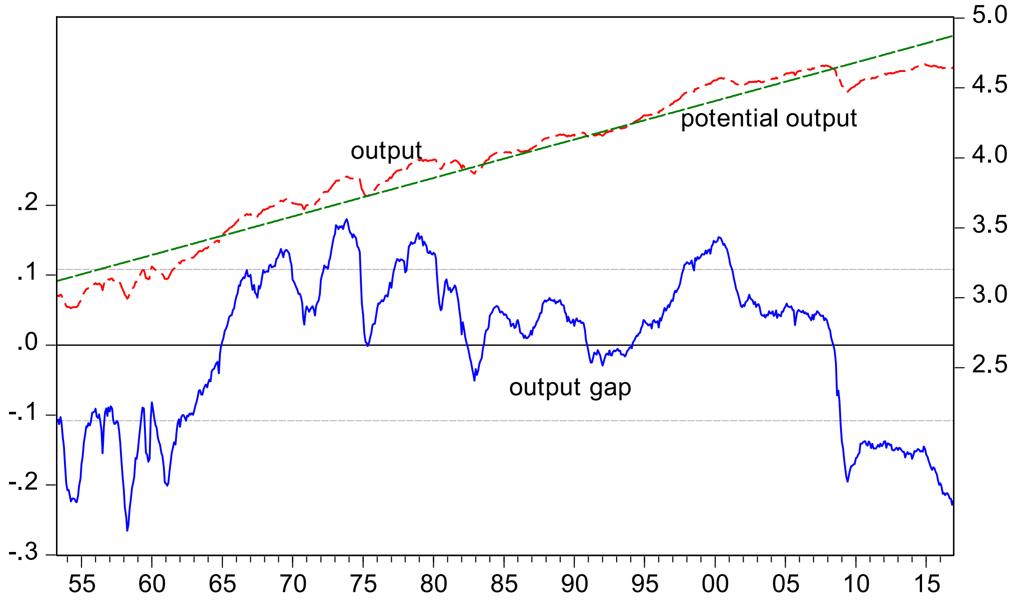
So the output gap is defined as a residual of following regression;

$$y_t = \underset{(0.00781)}{3.1175} + \underset{(0.00001)}{0.0022}\,t$$
$$R^2 = 0.956$$

where y_t is a industrial production index at time t and the number in parenthesis is a standard error.

This estimates trend growth in output over the periods to be about 2.2 per cent a year. See Figure 5.1. The output gap estimates represent several important historical business cycles. For instance, the output gap estimate shows a rapid decrease after the global financial crisis (2007) and an increase during IT bubble/boom period 2000-2005.

Figure 5.1: Linear Trend Estimate of the Output Gap



We then conducted the Augmented Dickey-Fuller (ADF) and Elliott-Rothenberg-Stock point optimal tests for a unit root checking of inflation, output gap and interest rate. We could not reject the null hypothesis that a variable has a unit root with a 1% level in every case. See Table 5.2. So we conclude that inflation, output gap and interest rate are all I(1).

Table 5.2: Unit root test results for model variable[1)]

variable	test type	include in test equation		
		none	constant	trend and constant
output gap	ADF[1)]	0.075	0.404	0.812
	Elliott-Rothenberg-Stock point optimal [2)]	-	5.535	20.12
inflation	ADF	0.107	0.035	0.099
	Elliott-Rothenberg-Stock point optimal	-	2.491	6.515
interest rate	ADF	0.255	0.278	0.388
	Elliott-Rothenberg-Stock point optimal	-	4.430	14.82

Note: 1) P-value for null hypothesis: the variable has a unit root.

2) P-statistic for null hypothesis: the variable has a unit root.

2) Test critical values for 1% level are 1.99 (when a constant is included in test equation) and 3.96 (when trend and constant are included in test equation) according to Elliott-Rothenberg-Stock (1996, Table 1).

For the construction of a VAR model of inflation, output gap and interest rate, the VAR lag order was set following Akaike and Schwartz information criterions. See Table 5.2 for the computed two criterions where the lag 3 has a common minimum value in two criterions. Therefore we select the lag order 3 and will consider a VAR (3) model for the upcoming analyses.

Table 5.3 Computed information criterions by the VAR order[1)]

order	1	2	3	4	5	6
AIC	-4.392	-5.305	<u>-5.392</u>	-5.382	-5.375	-5.374
SIC	-4.858	-5.176	<u>-5.207</u>	-5.142	-5.079	-5.022

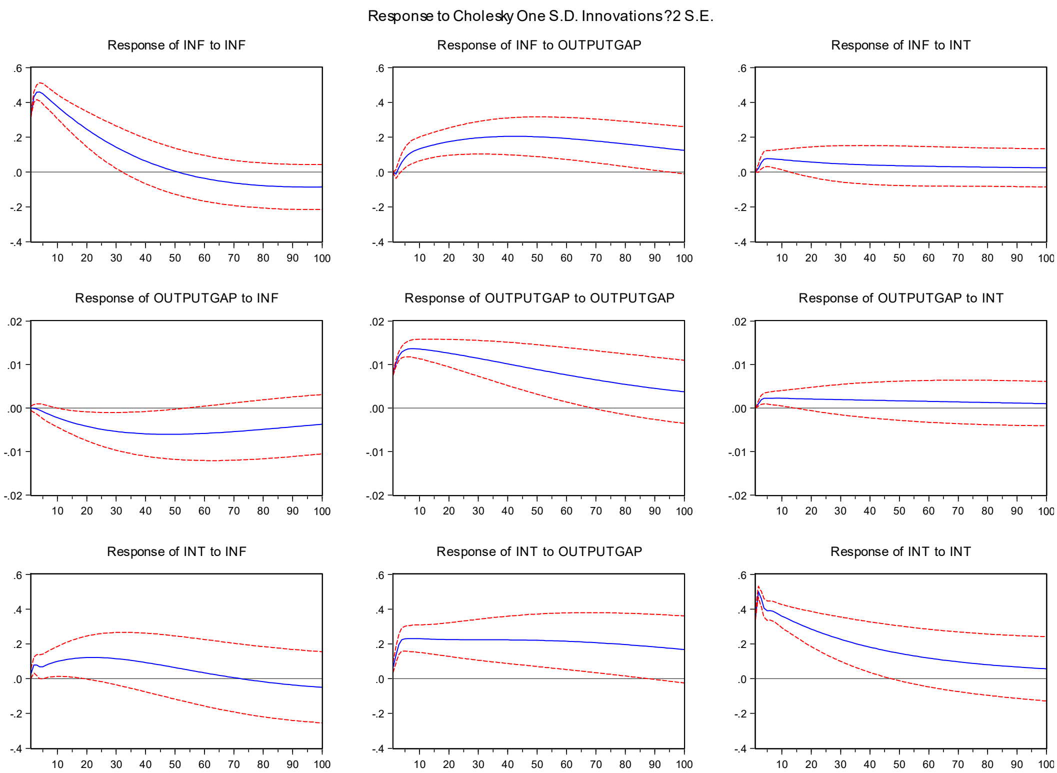
Note: 1) A constant is added.

We then conduct an impulse response analysis to check whether output gap affects to the interest rate significantly as New Keynesian frame work implied.[13] See Figure 5.4 for this analysis result using VAR(3) model of inflation, output gap and interest rate. In there, we can see that the impulse of output gap induces a significant response of interest rate during a substantially long time (around 90 months). That feature was little changed even we select a different Cholesky ordering from current one as the inflation, output gap and interest rate (c.f., (2.9)). This result is probably related with the output gap increase representing economic recovery induces the increase of money demand, and that

[13] If we just consider Fisher equation, it is not clear how the output gap is involved in the determination of real interest rate.

again results in the increase of interest rate. So our modeling approach to include the output gap in the determination of real interest rate is empirically supported.

Figure 5.4: Impulse response analyses for VAR model of inflation, output gap and interest rate



Note 1) standard error of response is computed by Monte Carlo simulation.

2) 'INF' denotes inflation, 'INT' denotes interest rate.

Now we conduct Johansen co-integration test to check whether there is a cointegration vector in the model. The null hypothesis that 'the hypothesized number of the co-integration is zero' was rejected in VAR(3) model at 1% level. See Table 5.5. So we will later assume that there is a cointegration vector in the model.

Table 5.5: Johansen co-integration test results

		VAR(3) with a constant term		
	hypothesized number of co-integration	0	1	2
p-values 2)	Trace	0.0004	0.0322	0.0335
	Maximum Eigenvalue	0.0034	0.1024	0.0335

Note: 1) P-values for hypothesized number of the co-integration.

2) No-intercept and no-trend are included in the co-integration equation and test VAR

Following Table 5.6 shows the estimators of cointegrating coefficients that is normalized by the coefficient of interest rate. In there, we find that interest rate is positively related with the inflation and negatively related with the output gap in the long run. These correlation signs are coincided with standard economic theory; i.e., the inflation will increase the interest rate while the interest rate increase will decrease the output gap at least in the long run.

Table 5.6: Estimator of Johansen co-integration coefficients

	i_t	π_t	g_t
VAR(3)	1	-1.4942 (0.1087)	1.0875 (4.5160)

Note: 1) standard error in parentheses.

2) We assume that there is a cointegration vector.

For the robustness checking of the above estimator of Johansen co-integration coefficients, we also estimated a cointegration vector using Engle-Granger OLS estimation as:

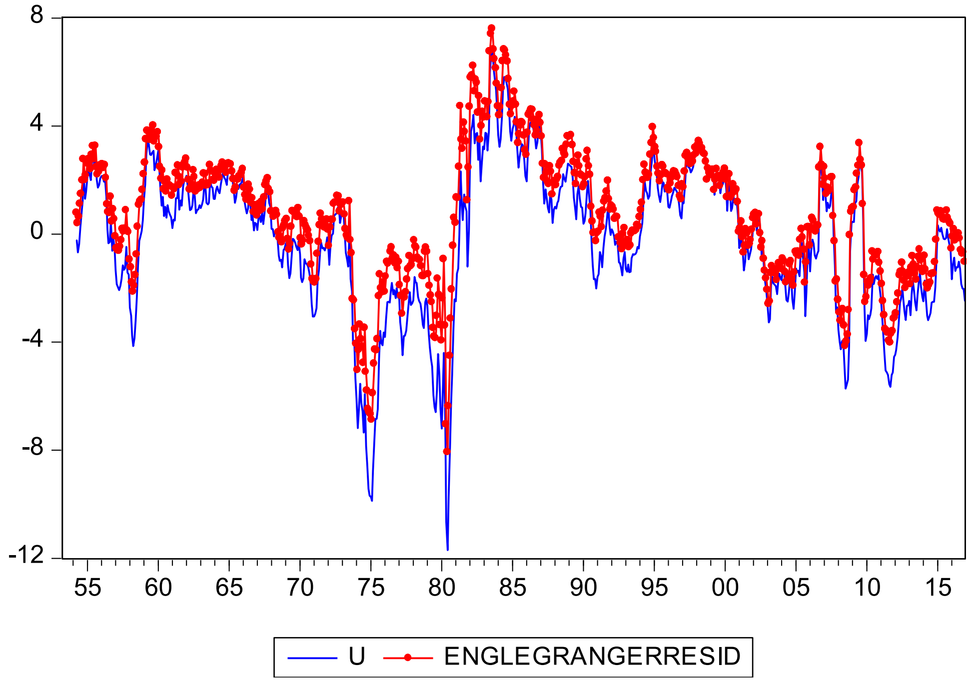
$$i_t = \underset{(0.0220)}{1.2057} * \pi_t + \underset{(0.9092)}{2.6457} * g_t$$

$$R^2 = 0.4302,$$

where a standard error is in parenthesis. Two estimation methods are providing different estimated coefficients. However, we can find that two methods result in the very similar cointegration errors. See Figure 5.7.

However we will use the estimators of Johansen co-integration coefficients for the inference of model (rather than Engle-Granger OLS estimator) because our model (3.3) is derived under Assumption 2.2 which is supposed in Johansen estimation of cointegration vector. See Figure 3.

Figure 5.7: Comparison of estimated cointegration error by Johansen and Engle-Granger methods



Note: 1) u denotes estimated cointegration error by Johansen method.

2) ENGLEGRANGERRESID denotes estimated cointegration error by Johansen method.

Then we conducted the stationarity test of real interest rate through a log likelihood (LR) test on the null hypothesis $\gamma^* = 0$ (c.f., Proposition 2.4). We found that the null has been rejected at the 1% level when the hypothesized number of cointegration is one. See Table 5.8 for the test results. So we conclude that real interest rate is non-stationary based on (i) the cointegration test results in Table 5.5, (ii) the test results on the null hypothesis $\gamma^* = 0$ in Table 5.8 and (iii) the rational expectation hypothesis respecting Proposition 2.4.

Table 5.8: LR test results for the null H_0 : $\gamma^* = 0$ [1)]

Hypothesized No. of cointegrations	VAR(3) with a constant term
1	0.0053
2	0.0187

Note: Number is P-value.

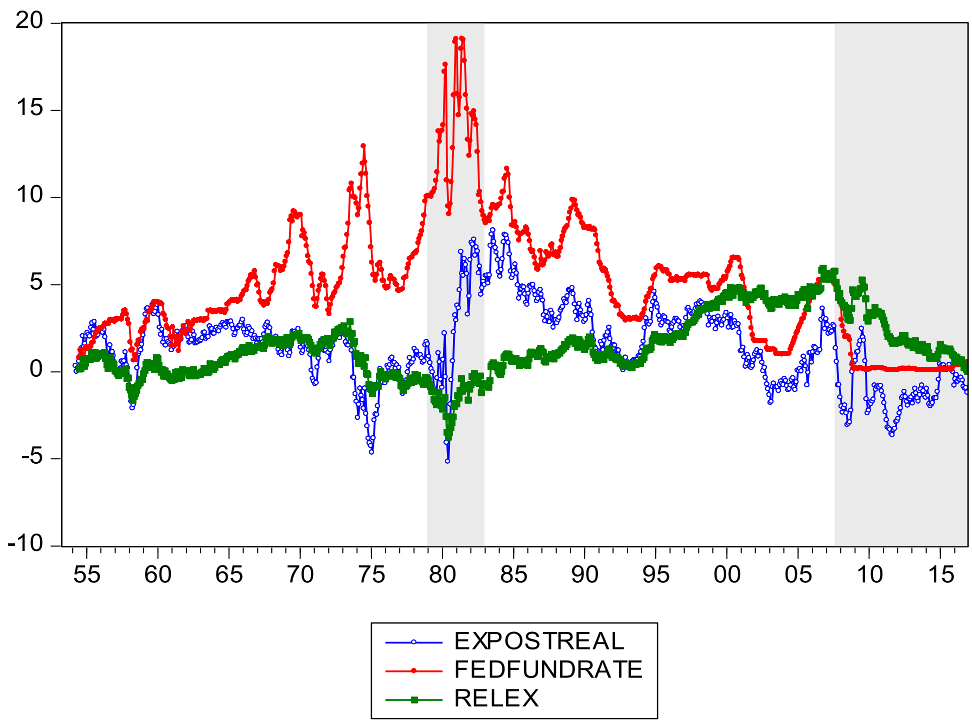
Finally, following Section 3, we estimated the decomposition of long run expectation of the nondeterministic part in real interest rate [suggested in (3.11)] as;[14]

$$\text{(5.1)} \quad \lim_{j\to\infty} E_t\left[r_{t+j} - \gamma^{*}{}'\psi_{01}(t+j)\right] = 0.3765 * \text{INTTREND}_t - 0.0320 * \text{INFTREND}_t + 5.7978 * \text{OUPUTTREND}_t$$

where $\eta_0 = 0$.

Now we estimated the RELEX of the real interest rate as $\hat{\gamma}^{*}{}'x_t$, that is a sum of interest rate, inflation and output gap trends. Then we graphed the federal fund rate, ex post real interest rate and RELEX (i.e., $\hat{\gamma}^{*}{}'x_t$) as in Figure 5.9. In there, we find that the non-stationary part from the real interest rate well explain the output gap variation different from the ex post real interest rate for two economically critical periods. At first, we can observe that relatively higher RELEX than the federal fund rate (close to zero) or the ex post real interest rate (even negative) just after global financial crisis (2007). This feature may explain why the US economy has not rapidly recovered (See Figure 5.1 for the negative and ongoing output gap after global financial crisis) even after aggressive quantitative easing by the Fed.[15] At second, we can also see that relatively lower RELEX than the federal fund rate or ex post real interest rate just after the second oil shock of 1978–79. This feature may also explain why the US economy has not been so damaged during this term (See Figure 5.1 for the positive output gap after the second oil shock).

Figure 5.9: Graphs for federal fund rate, ex post real interest rate and RELEX



[14] Eviews 7 and Gauss 7 were used for the computation. The codes are available on request.
[15] Unusual High RELEX may hinder to increase the investment.

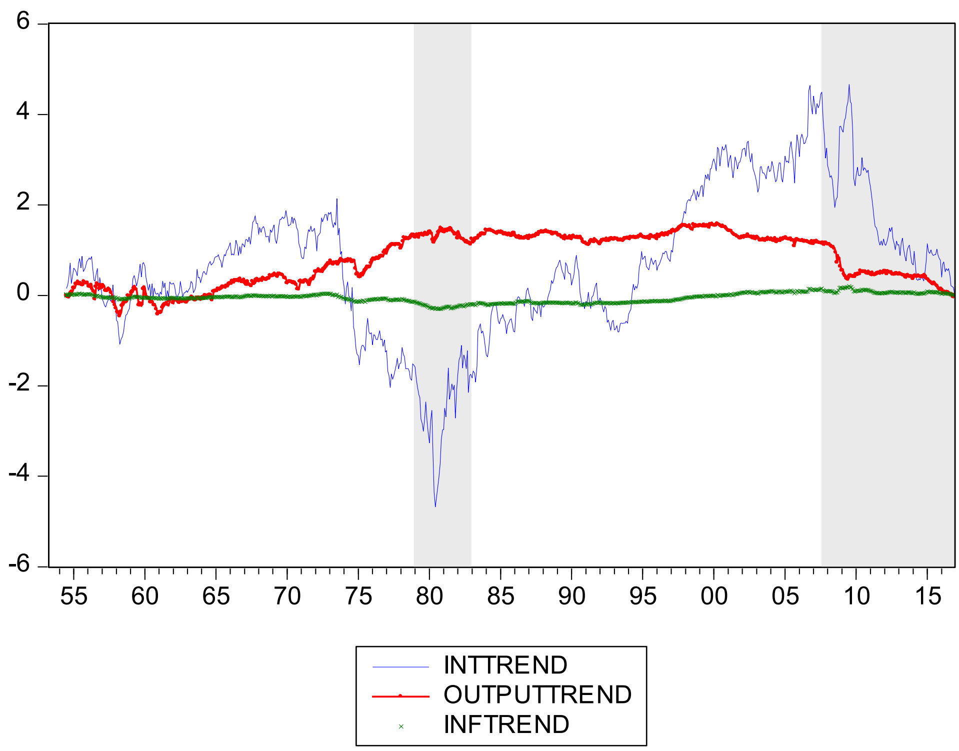
See Figure 5.9 fo the estimated individual contributions of the real interest rate's long run expectation (RELEX henceforth).

Next, we conducted the h_2 -test, as suggested in Section 3, in order to check the existence of INTTREND in real interest rate. We computed that $h_2 = 9.90$ and thus could reject the null hypothesis that 'there is not an INTTREND in the real interest rate' at the 1% level because the test statistic's 1% value is 9.21.

Using an estimator of INTTREND in (5.1), we may estimate the long run effect of monetary policy shock to the real interest rate consistently. For instance, 1% increase of federal fund target rate (inflation, outputgap) may approximately induce 0.4% (-0.03%, 5.8%) increase of RELEX respectively. It is remarkable that output gap shock has 15.4 times (=5.8%/0.4%) more powerful effect to the long run expectation of real interest rate than the monetary policy shock.

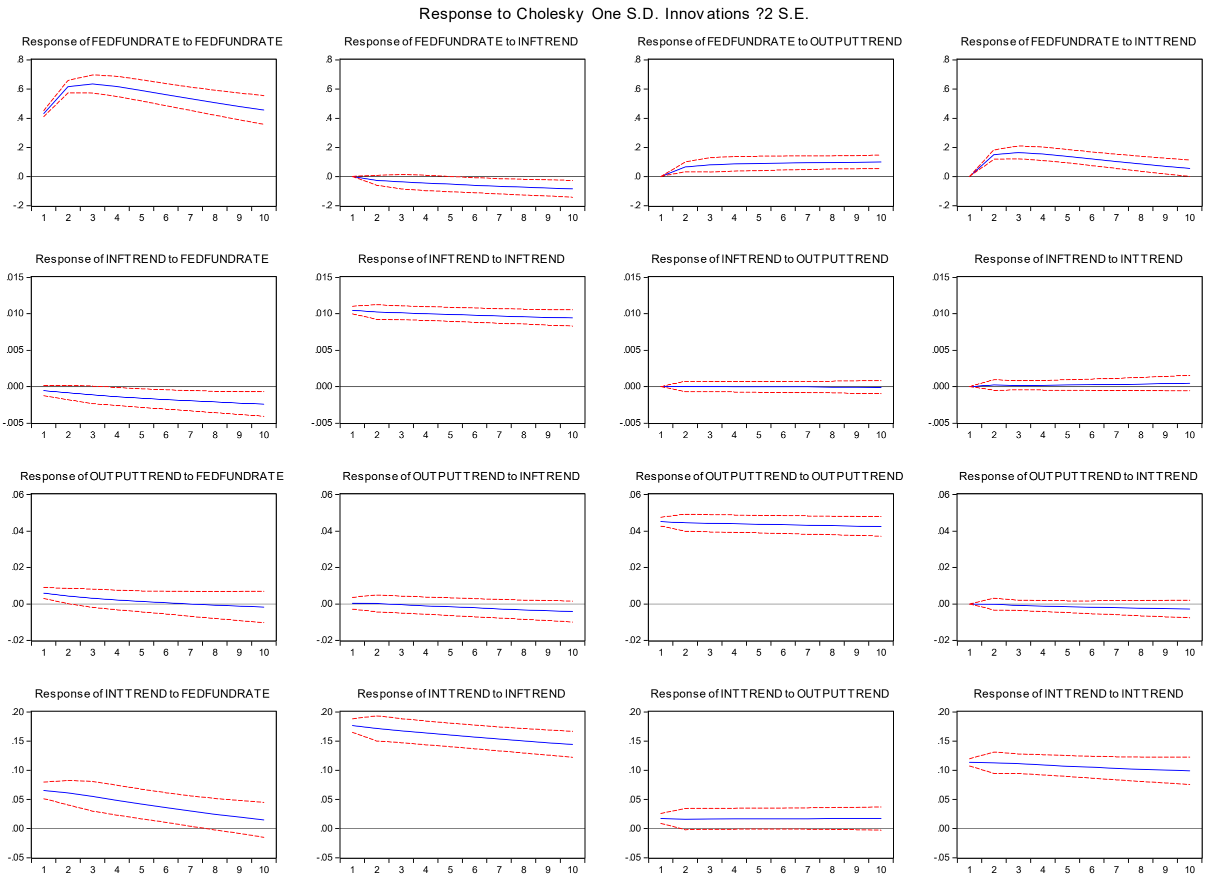
Now to figure out which component of RELEX is responsible for its high or low level, estimated three trends of RELEX are graphed as in Figure 5.10. In there, we can find that the INT has been record high after global financial crisis and record low after the second oil shock. So we may conclude that the INTTREND may explain the historical behavior of RELEX.

Figure 5.10: Estimated three trends of RELEX



Meanwhile, on the effectiveness of monetary policy, according to Woodford (2005), monetary policy is sufficiently effective just to the future/long run interest rate forecasting (or expectation) rather than instant interest rate change in the market. So we conducted the impulse response analysis using a VAR model of the federal fund rate and these trends of RELEX. It is to check how the monetary policy shock affects to the three trends of RELEX in the real interest rate. See Figure 5.11 for the results. We can see the federal fund rate increase does significantly affect to the INTTREND in RELEX. Especially, 1% increase of federal fund rate induces 0.05% increase of the INTTREND for about 7 months. That means monetary policy is effective to the future/long run interest rate expectation.

Figure 5.11: Impulse response analysis of VAR model with federal fund rate and the components of RELEX



Note 1) standard error of response is computed by Monte Carlo simulation.

2) 'INFTREND' denotes inflation shock trend, 'INTTREND' denotes interest rate shock trend and 'OUTPUTTREND' denotes outputgap shock trend respectively.

5. Conclusion

It has been revealed that the monetary policies (including zero target interest rate and quantative easing) in the US and other advanced economy countries seems to be insufficient to recover the economy from the depression after global financial crisis. However the reason of this failure of monetary policies has not fully understood yet considering the importance of monetary policy to the economy.

In this regard, this paper first introduced an augmented non-stationarity test of real interest rate within a cointegrated VAR model of interest rate, inflation and output gap reflecting New Keynesian frame work. Suggesting test additionally check whether the cointegration coefficients of inflation and output gap are 1 and 0 over conventional cointegration test.

We showed an interest rate shock trend including monetary policy shock (INTTREND) may be extracted from a non-stationary real interest rate using Beveridge-Nelson decomposition. We suggest a test to check the existence of INTTREND in the real interest rate and showed that a long run effect of monetary policy shock to the real interest rate may be estimated consistently. According to empirical analyses using the monthly US data, we found that suggested decomposition of real interest rate might be helpful to understand the long run role of monetary policy to the economy focusing on the essential role of real interest rate.

So the main differences of our approaches comparing with conventional ones are twofold. At first, we extend the real interest rate from just a 'black box' definition (interest rate minus expected inflation) to a variable determined by a New Keynesian behavioral system. In this procedure, the output gap has been added as a determinant of the real interest rate. At second, we derive a concept of long run expectation of real interest rate, which explicitly introduces the expectation that is probably related with the market psychology.

We agree that further researches on the variation of New Keynesian frame work seem to be interesting.

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