DANKOOK UNIVERSITY

RIFI

The Research Institute of Future Industry

RIFI Working Paper

http://cms.dankook.ac.kr/web/rifi

RIFI Working Paper 17-01

May Monetary Policy Affect to Long Run Expextation of Non-Stationary Real Interest Rate ?

by

Yun-Yeong Kim

June, 2017

The Research Institute of Future Industry
DANKOOK UNIVERSITY
152, Jukjeon-ro, Sujigu, Yongin-si,
Gyeonggi-do, 16890, Korea

Does vector autoregressive generalized Solow growth model explain business cycles ?

by

Yun-Yeong Kim*

Abstract

In this paper, we decompose the I(1) trends of output into capital, labor and technology shock to compute their long run contributions to the output. For this, we generalize standard *Solow growth model* (SGM) by a *vector autoregressive* (VAR) form of log transformed capital, labor and output. In our approach, (i) unique shocks to the inputs (capital and labor) are allowed and (ii) inputs might be simultaneously or dynamically affected by the output. Under our structure, the per capita output may not converge to a fixed value as in SGM and it rather can be a random walk process in the long run with or without a time trend. Further the existence of I(1) technology shock trend in the output that is independent with the capital or labor shock trends might be statistically tested. Using the yearly US data after 1950, we found the sudden drops of capital shock trends may explain the causes of main business cycles; i.e., the early 1990's recession and the global financial crisis. Finally, we found that (i) we can not statistically reject the null hypothesis that a technology shock trend does not affect the output in the long run and (ii) the capital shock trend induces longer response of the output than labor or technology shock trends.

JEL Classification Number: E40

Keywords: Solow growth model; business cycles; input shocks; VAR model, trend decomposition

* Department of International Trade, Dankook University, 126, Jukjeon-dong, Yongin-si, Gyeonggi-do 448-701, Korea, Email: yunyeongkim@dankook.ac.kr.

1. Introduction

It seems that we still do not understand the cause of global financial crisis (GFC henceforth); i.e., what drives the output down so rapidly during the GFC. For instance, see Ohanian (2010) for this issue.[1] According to the real business cycle (RBC) models that explain the business cycles using the technology shock estimated by Solow residual (SR), the cause of the GFC should be explained by the dramatic decrease of the SR.

However the SR confronts numerous objections from two opposite side of views; (i) it is not sufficient to explain the business cycles; c.f., Summers (1986) and Mankiw (1989), and (ii) it is over sufficient and one not filtered the non-technology noises. For instance, Bernanke and Parkinson (1992) find that the SR moves just as much with output in the Great depression, even though it does not seem that the depression was caused by technology progress. Hall (1988) shows that movements in the SR are correlated with the political party of the president, changes in military purchases, and oil-price movements; these variables do not seem to affect technology significantly.

Two explanations are possible for these problems of the SR. First the SR may not be a correct measure of technology shocks because of its inappropriate estimation procedure. Second, the technology shock may not be a unique source driving the business cycles.

For the first point, we have to note the constant return to scale (CRS henceforth) is restricted in estimation of SR, where the CRS itself need to be empirically tested. If the CRS is violated de facto, then the SR might include inputs in there. Further, even if the CRS assumption is correct, the SR may not be a pure technology shock series if it is simultaneously correlated with the capital or labor; i.e., there might be an endogeneity bias.

For the second point, we have to note that we need to add these inputs (capital and labor) shocks into a model to explain the business cycles. It is because capital or labor as well as the technology shocks may happen in the economy. There can often be labor shock like immigration and capital shock like SOC (social capital) expanding by the government while those are suspected to affect the business cycle. See Mankiw, Romer and Weil (1992) supporting this view.

To respond these problems and critiques, we suggest to generalize standard Solow (1957) growth model (SGM) by a vector autoregressive (VAR) form of log transformed capital, labor and output a la Sims (1980). In our approach, (i) unique shocks to the inputs are allowed and (ii) inputs might be

[1] "More broadly, neoclassical business cycle research has established a significant base of knowledge on how model economies respond to a variety of abstract shocks. However, we know less about the specific sources and nature of these shocks, particularly about cyclical distortions to productivity and to labor markets. Thus, we do not as yet have satisfactory answers to a number of questions, including why labor market deviations were so much larger in the U.S. economy in the 2007-2009 recession than in earlier recessions, why labor market deviations seem so much larger in the United States than in other high-income countries, why productivity deviations seem to play such a large role in other high-income countries than in the United States, how to model real-world financial and policy events for determining their impact on the economy, and why macroeconomic weakness continued for so long after the worst of the crisis passed. (pp. 63-64)"

simultaneously or dynamically affected by the output. Then we decompose the I(1) trends of output into capital, labor and technology shock to compute their long run contributions to the output.

Under our structure, the per capita output may not converge to a fixed value as in SGM and it rather can be a random walk process. Further the existence of I(1) technology shock trend that is independent with the capital or labor shock trends might be statistically tested.

The rest of this paper proceeds as follows. Section 2 discusses the VAR representation of Solow growth model. Section 3 focuses on the extraction of long-run trends in output and introduces the inference for these trends. The empirical analysis result for the US data is discussed in Section 4. Section 5 contains the conclusion.

2. VAR representation of Solow growth model

In this section, we first show Solow growth model may be transformed into a vector autoregressive (VAR) form of log transformed variables. For this, suppose a Cobb-Douglas production function as

(2.1) $Y_t = K_t^{\alpha} L_t^{1-\alpha}$

where Y_t is an output, K_t is a capital, L_t is a labor at time t, respectively. The capital accumulation equation is given as

(2.2) $K_{t+1} = K_t + \dot{K}_t$

where

(2.3) $\dot{K}_t = sY_t - \delta K_t$

where δ denotes a depreciation rate and *s* denotes a saving rate, respectively. Note $\dot{K}_t$ denotes the net increment of capital stock from t to t+1. The labor is assumed as growing at the constant rate *n*;

(2.4) $(L_{t+1} - L_t)/L_t = n$.

Now we approximate (2.1), (2.2) and (2.4) as a VAR model at a stationary state after log transformed the model variables. At first, we log transform (2.1) and get

(2.5) $\ln Y_t = \alpha \ln K_t + (1-\alpha)\ln L_t$

At second, we log transform (2.2) and get

(2.6) $\ln K_{t+1} = \ln(K_t + \dot{K}_t) = \ln K_t + \dot{K}_t / K_t + \nu_{t+1}$

using Taylor approximation at K_t for the second equation and ν_{t+1} is an approximation error. Let k^* be a value of $k_t = K_t / L_t$, which satisfies $sk_t^{\alpha} = (n+\delta)k_t$ where actual investment (sk_t^{α}) is equal to break-even investment($(n+\delta)k_t$). If k_t is not initial zero, it converges to k^* regardless of where k_t starts under standard regularity conditions. See Romer (2000). Then we may write (2.6) as:

(2.7) $\ln K_{t+1} = \ln K_t + s(k^*)^{\alpha-1} - \delta + e_{Kt+1}$

using (2.3) because

$$\dot{K}_t / K_t = sY_t / K_t - \delta = s(k_t)^{\alpha-1} - \delta = s(k^*)^{\alpha-1} + e_{Kt+1}$$

where $e_{Kt+1} = s[(k_t)^{\alpha-1} - (k^*)^{\alpha-1}] + \nu_{t+1}$. Finally, we may approximate (2.4) as;

(2.8) $\ln L_{t+1} - \ln L_t = n$.

Consequently, we may construct a structural VAR(1) model of observable variables with (2.5), (2.7) and (2.8) as:

(2.9) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\alpha & \alpha-1 & 1 \end{pmatrix} Z_{t+1} = \begin{pmatrix} s(k^*)^{1-\alpha} - \delta \\ n \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} Z_t + \begin{pmatrix} e_{Kt+1} \\ 0 \\ 0 \end{pmatrix}$,

where $Z_{t+1} \equiv \begin{pmatrix} \ln K_{t+1} \\ \ln L_{t+1} \\ \ln Y_{t+1} \end{pmatrix}$.

Note (2.9) might be written as a reduced form VAR model as

(2.10) $Z_{t+1} = \Pi_0 + \Pi_1 Z_t + \varepsilon_{t+1}$,

where

$\Pi_0 = \begin{pmatrix} s(k^*)^{1-\alpha} - \delta \\ n \\ \alpha[s(k^*)^{1-\alpha} - \delta] + (1-\alpha)n \end{pmatrix}$, $\Pi_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \alpha & 1-\alpha & 0 \end{pmatrix}$ and $\varepsilon_{t+1} = \begin{pmatrix} e_{Kt+1} \\ 0 \\ \alpha e_{Kt+1} \end{pmatrix}$ because

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\alpha & \alpha-1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \alpha & 1-\alpha & 1 \end{pmatrix}.$$

Note the error term e_{Kt+1} in (2.9) may have memory on its lagged ones.[2] So we may generalize (2.9) as a structural VAR(q) model (henceforth VAR-SGM) as follows:

(2.11) $\Gamma Z_{t+1} = \Gamma_0 + \sum_{i=1}^{q} \Gamma_i Z_{t-i+1} + \zeta_{t+1}$

[2] e_{Kt+1} has an error of adjustment process that might be approximated by ARMA process.

where $\Gamma = \begin{pmatrix} \underset{2\times 2}{\Gamma_{11}} & \underset{2\times 1}{0} \\ \underset{1\times 2}{\Gamma_{21}} & \underset{1\times 1}{1} \end{pmatrix}$ and $\zeta_{t+1} \equiv (\zeta_{kt+1}, \zeta_{lt+1}, \zeta_{yt+1})'$ is a vector of unexpected shocks for capital, labor and output respectively.

For instance, the shock to the capital ζ_{kt+1} may include the interest rate change by the monetary policy. The shock to the labor ζ_{lt+1} may include immigration and birth rate change. ζ_{yt+1} may represnet the technology shock to the output.

Now we assume,

Assumption 2.1 $(\zeta_t)_{t=1}^{n}$ *is an independent and identically distributed sequence with a distribution as;*[3]

$$\zeta_t \sim \left[0, \begin{pmatrix} \Sigma_l & 0 & 0 \\ 0 & \Sigma_k & 0 \\ 0 & 0 & \Sigma_y \end{pmatrix} \right].$$

Several remarks are worthy of note on the VAR-SGM (2.11). First, note the coefficient matrix Γ is assumed as block triangular. It reflects that the labor and capital shocks do not respond to the output shock instantly because those inputs need time to adjust after a shock. For instance, to build factory and labor force movement usually take time.

Second, we allow lagged outputs may affect the inputs dynamically in (2.11). For instance, the capital formation may depend on the lagged output through acceleration principle. Further, the fast growing output may attract immigration and thus induce the increase of labor supply.[4]

Now note a reduced form VAR model (2.11) is given as

(2.12) $Z_{t+1} = \Pi_0 + \sum_{i=1}^{q} \Pi_i Z_{t-i+1} + \varepsilon_{t+1}$

or

$$\Delta Z_{t+1} = \Pi_0 + \Phi Z_t + \sum_{i=1}^{q-1} \Phi_i \Delta Z_{t-i+1} + \varepsilon_{t+1} ,$$

[3] This iid error assumption might be generalized to include a martingale difference sequence.

[4] See Romer (2000, p163) "employment and hours are strongly procyclical-that is, they move in the same direction as aggregate output."

where $\Pi_i \equiv \Gamma^{-1}\Gamma_i \equiv \begin{pmatrix} \underset{2\times 2}{\Pi_{11i}} & \underset{2\times 1}{\Pi_{12i}} \\ \underset{1\times 2}{\Pi_{21i}} & \underset{1\times 1}{\Pi_{22i}} \end{pmatrix}$ is a 3×3 coefficient matrix for $i = 1, 2, \cdots, q$, $\Delta Z_t = Z_t - Z_{t-1}$,

$\Phi = \sum_{i=1}^{q} \Pi_i - I_3$, $\Phi_i = -(\Pi_{i+1} + \Pi_{i+2} + \cdots + \Pi_q)$ and

$\varepsilon_{t+1} \equiv \Gamma^{-1}\zeta_{t+1} \equiv \begin{pmatrix} \underset{2\times 2}{\Gamma_{11}^{-1}} & \underset{2\times 1}{0} \\ \underset{1\times 2}{-\Gamma_{21}\Gamma_{11}^{-1}} & \underset{1\times 1}{1} \end{pmatrix} \begin{pmatrix} \zeta_{kt+1} \\ \zeta_{lt+1} \\ \zeta_{yt+1} \end{pmatrix} \equiv (\delta_{t+1}', \xi_{t+1})'$ where $\delta_{t+1} \equiv \Gamma_{11}^{-1} \begin{pmatrix} \zeta_{kt+1} \\ \zeta_{lt+1} \end{pmatrix}$ and

$\xi_{t+1} \equiv -\Gamma_{21}\delta_{t+1} + \zeta_{yt+1}$.

In this paper, we assume that Z_t is I(1). Then the co-integration is represented by the singularity restriction (Johansen, 1991) of the long-run impact matrix Φ, as follows:

Assumption 2.2 *Suppose that* $\Phi = \phi_1\phi_2'$, *where* $\phi_2 = (\underset{1\times 2}{-\gamma'}, 1)'$ *is a co-integration vector with* $\gamma \neq 0$[5] *where* ϕ_1 *and* ϕ_2 *are* 3×1 *vectors respectively.*

Note ϕ_2 represents the long run equilibrium relation between the output and inputs.

Then, in following section, we will conduct inference for the long run trends of capital, labor and technology in the output using Model (2.12).

3. Inference for long-run trends in output

To extract I(1) trends from the output, we consider a transformed VAR model of (2.12):

(3.1) $TZ_{t+1} = T\Pi_0 + \sum_{i=1}^{q} (T\Pi_i T^{-1}) TZ_{t-i+1} + e_{t+1}$,

where T is a transformation matrix with a co-integration vector as:

(3.2) $T \equiv \begin{pmatrix} I_2 & 0 \\ -\gamma' & 1 \end{pmatrix}$, $|T| \neq 0$,

where $x_t = (\ln K_t, \ln L_t)'$, $u_t = \ln Y_t - \gamma' x_t$, $TZ_t = (x_t', u_t)'$ and $e_t = T\varepsilon_t = (\delta_t, \xi_{\gamma t})'$, with $\xi_{\gamma t} \equiv \xi_t - \gamma'\delta_t$.

---

[5] The case when the number of cointegration vector is two may be similarly analyzed. In this paper, we do not restrict the co-integration vector β.

Remark 3.1 A co-integration relation might be exploited to deduce return to scale in the long run. For instance, suppose there is h-fold increase of capital and labor as hK_t and hL_t, then the co-integration equation

(3.3) $\ln Y_t = \gamma' x_t + u_t$

is changed as $\ln Y_t = \gamma'(1,1)\ln h + \gamma' x_t + u_t$.

We may readily check $\gamma'(1,1)$ determines the degree of return to scale in the long run where a co-integration error u_t is I(0). So if $\gamma'(1,1) = 1$, then there is a CRS in the long run. We may test on this restriction of co-integration coefficient using a log likelihood (LR) test in Johansen (1995). ■

Then, following Kim (2012), we may transform the model (3.1) into a VAR model of the stationary variable under Assumption 2.2 as follows:

(3.4) $$\begin{pmatrix} \Delta x_{t+1} \\ u_{t+1} \end{pmatrix} = \Psi_0 + \sum_{i=1}^{q} \Psi_i \begin{pmatrix} \Delta x_{t-i+1} \\ u_{t-i+1} \end{pmatrix} + \begin{pmatrix} \delta_{t+1} \\ \xi_{\gamma t+1} \end{pmatrix},$$

with $\Psi_{11q} = 0$ and $\Psi_{21q} = 0$ where

$$\Psi_0 \equiv \begin{pmatrix} \underset{2\times1}{\Psi_{01}} \\ \underset{1\times1}{\Psi_{02}} \end{pmatrix} \equiv T\Pi_0 \quad \text{and} \quad \Psi_i \equiv \begin{pmatrix} \underset{2\times2}{\Psi_{11i}} & \underset{2\times1}{\Psi_{12i}} \\ \underset{1\times2}{\Psi_{21i}} & \underset{1\times1}{\Psi_{22i}} \end{pmatrix} \quad \text{for } i = 1,2,...,q.$$

Then, if invertible, Model (3.4) may also be written as a vector moving average form:

(3.5) $$\begin{pmatrix} \Delta x_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} \psi_{01} \\ \psi_{02} \end{pmatrix} + \begin{pmatrix} \theta_{11}(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{pmatrix} \begin{pmatrix} \delta_{t+1} \\ \xi_{\gamma t+1} \end{pmatrix},$$

where L is a time lag index and

$$[I_3 - \sum_{i=1}^{q} \Psi_i]^{-1} \Psi_0 = \begin{pmatrix} \underset{2\times1}{\psi_{01}} \\ \underset{1\times1}{\psi_{02}} \end{pmatrix} \quad \text{and}$$

$$\begin{pmatrix} \theta_{11}(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{pmatrix} \equiv \begin{pmatrix} I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i & -\sum_{i=1}^{q} \Psi_{12i} L^i \\ -\sum_{i=1}^{q-1} \Psi_{21i} L^i & 1 - \sum_{i=1}^{q} \Psi_{22i} L^i \end{pmatrix}^{-1},$$

where

$$\theta_{11}(L) \equiv [(I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i + \sum_{i=1}^{q-1} \Psi_{12i} L^i)(1 - \sum_{i=1}^{q} \Psi_{22i} L^i)^{-1} (\sum_{i=1}^{q-1} \Psi_{21i} L^i)]^{-1}$$
$$\equiv \sum_{i=0}^{\infty} L^i \theta_{11i}$$

and

$$\theta_{12}(L) \equiv (I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i)^{-1} (\sum_{i=1}^{q} \Psi_{12i} L^i) \times$$
$$[1 - \sum_{i=1}^{q} \Psi_{22i} L^i - (\sum_{i=1}^{q-1} \Psi_{21i} L^i)(I_2 - \sum_{i=1}^{q-1} \Psi_{11i} L^i)^{-1} (\sum_{i=1}^{q} \Psi_{12i} L^i)]^{-1}$$
$$\equiv \sum_{i=0}^{\infty} L^i \theta_{12i}.$$

Then, from (3.5), the Beveridge-Nelson (1981) decomposition of x_t is defined as[6]

(3.6) $$x_t = x_0 + \sum_{s=1}^{t} \Delta x_s = x_0 + \theta_{11}(1) \sum_{s=1}^{t} \delta_s + \theta_{12}(1) \sum_{s=1}^{t} \xi_{\gamma s} + \psi_{01} t + \eta_t - \eta_0 ,$$

where $\Delta x_t = \theta_{11}(L)\delta_t + \theta_{12}(L)\xi_{\gamma t}$ and $\sum_{t=1}^{\infty} t^{1/2} | \theta_{1it} | < \infty$ for i=1,2 and; η_t is a stationary process.

Now to extract the technology shock trend that is orthogonal with the input shock trend as,

(3.7) $$x_t = x_0 + [\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}'] \sum_{s=1}^{t} \delta_s + \theta_{12}(1) \sum_{s=1}^{t} \tilde{\xi}_s + \psi_{01} t + \eta_t - \eta_0$$

where $\bar{\gamma} \equiv (E\delta_t\delta_t')^{-1} E\delta_t\xi_{\gamma t}$ and $\tilde{\xi}_t \equiv \xi_{\gamma t} - \bar{\gamma}'\delta_t$. Note $\bar{\gamma}$ is a non-zero population projection coefficient that satisfies $E\tilde{\xi}\delta_t = 0$. Thus the shock $\tilde{\xi}_t$ is not correlated with the shock δ_t (and is independent of the shock δ_t under the normality of ε_t).

Then we show the shock $\tilde{\xi}_t$ is equivalent with the structural shock to the output in (2.11).

Lemma 3.2 *Suppose* Assumption 2.1 *holds*. *Then* $\tilde{\xi}_t = \zeta_{yt}$.

Proof: Note

(3.8) $$\tilde{\xi}_t \equiv \xi_{\gamma t} - \bar{\gamma}'\delta_t = \xi_t - \gamma'\delta_t - (\lambda - \gamma)'\delta_t = \xi_t - \lambda'\delta_t$$

because $\bar{\gamma} \equiv (E\delta_t\delta_t')^{-1} E\delta_t\xi_{\gamma t} = (E\delta_t\delta_t')^{-1} E\delta_t(\xi_t - \delta_t'\gamma) = \lambda - \gamma$ where

(3.9) $$\lambda \equiv (E\delta_t\delta_t')^{-1} E\delta_t\xi_t = (E\delta_t\delta_t')^{-1} E\delta_t\left[-\Gamma_{21}\delta_t + \zeta_{yt}\right] = -\Gamma_{21}'$$

from the definition in (2.11) for the second equality and $E\delta_t\zeta_{yt} = 0$ from Assumption 2.1 for the third equality. Therefore we get the claimed result because

$$\tilde{\xi}_t = \xi_t - \lambda'\delta_t = \xi_t + \Gamma_{21}\delta_t = \zeta_{yt}$$

from (2.12), (3.8) and (3.9). Q.E.D.

Now, from Lemma 3.2, we may rewrite (3.7) as

(3.10) $$x_t = x_0 + [\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}'] \sum_{s=1}^{t} \delta_s + \theta_{12}(1) \sum_{s=1}^{t} \zeta_{ys} + \psi_{01} t + \eta_t - \eta_0$$

[6] For reference, see Hamilton (1994, *pp*545-546).

where $\sum_{s=1}^{t}\zeta_{ys}$ is a technology shock trend (TECHSHOCKTREND henceforth) and $\sum_{s=1}^{t}\delta_t$ is the trend of capital and labor shocks (CAPITALSHOCKTREND, LABORSHOCKTREND henceforth). So three trends are independent with each other under Assumption 2.1 and Lemma 3.2 under the normality assumption.

Now we show the output has a random walk stochastic trend without a time trend if $\gamma'\psi_{01} = 0$ as:

Proposition 3.3 *Suppose that (i)* u_t *is a mean zero process,* (*ii*) $\psi_{01} = \psi_{02} = 0$[7] *and* (*iii*) $\gamma' x_0 = \gamma'\eta_0 = 0$. *Then we obtain*

(3.11) $\lim_{j\to\infty} E_t \ln Y_{t+j} = \gamma'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s + \gamma'\theta_{12}(1)\sum_{s=1}^{t}\zeta_{ys}$

Proof: Note that

$$\lim_{j\to\infty} E_t \ln Y_{t+j} = \lim_{j\to\infty} E_t\left[u_{t+j} + \gamma'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t+j}\delta_s + \gamma'\theta_{12}(1)\sum_{s=1}^{t+j}\zeta_{ys} + \gamma'\eta_{t+j}\right]$$

$$= \gamma'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s + \gamma'\theta_{12}(1)\sum_{s=1}^{t}\zeta_{ys}$$

from (3.3) and (3.10) for the first equality and because $\lim_{j\to\infty} E_t u_{t+j} = 0$ and $\lim_{j\to\infty} E_t \gamma'\eta_{t+j} = 0$ where (u_t) and $(\gamma'\eta_t)$ are the stationary process of mean zero from assumption. Q.E.D.

Note that a long run expected marginal productivities of inputs and technology are given as;

(a) $\dfrac{\partial \lim_{j\to\infty} E_t \ln Y_{t+j}}{\partial \delta_t} = \gamma'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']$ and

(b) $\dfrac{\partial \lim_{j\to\infty} E_t \ln Y_{t+j}}{\partial \zeta_{yt}} = \gamma'\theta_{12}(1)$

From (3.11).

Further note the labor is decomposed as

[7] We may test this assumption using a t-test for the null hypothesis of $\Psi_0 = 0$ from equations (3.4) and (3.5) as long as $I_3 - \sum_{i=1}^{q}\Psi_i$ is not singular, that has a standard limit distribution because of the super-consistency of the cointegration vector (c.f., Kim 2014). See Table 4.8 for the test result.

(3.12) $\ln L_t = (0,1)'x_0 + (0,1)'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s + (0,1)'\theta_{12}(1)\sum_{s=1}^{t}\zeta_{ys} + (0,1)'\psi_{01}t + (0,1)'(\eta_t - \eta_0)$

from (3.10). Therefore a per capita output is given as

(3.13) $\ln(Y_t / L_t) = u_t + \gamma^{*}{}'x_0 + \gamma^{*}{}'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s + \gamma^{*}{}'\theta_{12}(1)\sum_{s=1}^{t}\zeta_{ys} + \gamma^{*}{}'\psi_{01}t + \gamma^{*}{}'(\eta_t - \eta_0)$

From (3.12) where $\gamma^{*} \equiv \gamma - (0,1)$. Therefore we get a long run per capita output is given as;

(3.14) $\lim_{j\to\infty} E_t[\ln(Y_{t+j} / L_{t+j}) - \gamma^{*}{}'\psi_{01}(t+j)] = \gamma^{*}{}'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s + \gamma^{*}{}'\theta_{12}(1)\sum_{s=1}^{t}\zeta_{ys}$.

Remark 3.4 Several remarks on the difference of our approach with conventional growth theory are noteworthy. First, note the long run per capita output is expected as a constant $k^{*\alpha}$ in the SGM, which is a special case in a generalized SGM where $\delta_s = 0$ and $\theta_{12}(1) = 0$.[8]

Second, if per capita output follows a random walk in the long run, then the convergence hypothesis that poor countries tend to grow faster than the rich countries need to be reconsidered. It is because convergence hypothesis partly depends on that the SGM predicts countries converge to their balanced growth paths.[9]

Finally, note just technology shock does matter in RBC models. Our approach allows that input shocks may be negative even if there is a non-negative technology shock, that may explain the depression. ■

To estimate the above trends, we take the following steps (c.f., Kim; 2014, 2016, 2017):

1. Estimate Π_i as $\hat{\Pi}_i$ for all $i = 1,2,\cdots,q$ from the VAR model (2.12) and get the residual $(\hat{\delta}_t, \hat{\xi}_t)'$; $t = 1,2,\cdots,n.$
2. Estimate the co-integration coefficient β as $\hat{\beta} = (-\hat{\gamma}',1)$ by Johansen (1991) and estimate γ as $\hat{\gamma}$.
3. Estimate λ, $\bar{\gamma}$ and $\tilde{\xi}_t$ as $\hat{\lambda} \equiv (\sum_{s=1}^{n}\hat{\delta}_s\hat{\delta}_s')^{-1}\sum_{s=1}^{n}\hat{\delta}_s\hat{\xi}_s$, $\hat{\bar{\gamma}} = \hat{\lambda} - \hat{\gamma}$ and $\hat{\tilde{\xi}}_t = \hat{\xi}_t - \hat{\lambda}'\hat{\delta}_t$.
4. Estimate the co-integration error u_t as $\hat{u}_t = \hat{\beta}'z_t$.
5. Run an OLS regression (3.4) replacing u_t into $\hat{u}_t$ to get the estimators of Ψ_i as $\hat{\Psi}_i$ for $i = 0,1,2,\cdots,q.$
6. Compute the estimator of $\theta_{11}(1)$ and $\theta_{12}(1)$ in (3.5) as

---

[8] It means that there are not input shocks ($\delta_s = 0$) and we will later show that the co-integration error u_t does not block Granger cause to the fundamental change Δx_t in (3.4) ($\theta_{12}(1) = 0$).

[9] See Baumol(1986), De Long(1988) and Mankiw, Romer and Weil (1992) for this issue.

$$\hat{\theta}_{11}(1) = [I_2 - \sum_{i=1}^{q-1}\hat{\Psi}_{11i} + (\sum_{i=1}^{q-1}\hat{\Psi}_{12i})(1 - \sum_{i=1}^{q}\hat{\Psi}_{22i})^{-1}(\sum_{i=1}^{q-1}\hat{\Psi}_{21i})]^{-1};$$

$$\hat{\theta}_{12}(1) = (I_2 - \sum_{i=1}^{q-1}\hat{\Psi}_{11i})^{-1}(\sum_{i=1}^{q}\hat{\Psi}_{12i})\times$$
$$[1 - \sum_{i=1}^{q}\hat{\Psi}_{22i} - (\sum_{i=1}^{q-1}\hat{\Psi}_{21i})(I_2 - \sum_{i=1}^{q-1}\hat{\Psi}_{11i})^{-1}(\sum_{i=1}^{q}\hat{\Psi}_{12i})]^{-1}.$$

7. Estimate the trends at time t as:

$$\hat{\gamma}'[\hat{\theta}_{11}(1) + \hat{\theta}_{12}(1)\hat{\bar{\gamma}}']\sum_{s=1}^{t}\hat{\delta}_s \quad \text{and} \quad \hat{\gamma}'\hat{\theta}_{12}(1)\sum_{s=1}^{t}\hat{\tilde{\xi}}_s .$$

Now we suggest testing the existence of TECHSHOCKTREND in the output. Note the TECHSHOCKTREND does not exist if $\theta_{12}(1) = 0$ from (3.11) when $\gamma \neq 0$. Note the null hypothesis $\theta_{12}(1) = 0$ holds if and only if

(3.15) $\sum_{i=1}^{q}\underset{2\times1}{\Psi_{12i}} = 0$,

from (3.5) where $(I_2 - \sum_{i=1}^{q-1}\Psi_{11i})^{-1}$ and $[1 - \sum_{i=1}^{q}\Psi_{22i} - (\sum_{i=1}^{q-1}\Psi_{21i})(I_2 - \sum_{i=1}^{q-1}\Psi_{11i})^{-1}(\sum_{i=1}^{q}\Psi_{12i})]^{-1}$ are not singular. It is noteworthy that the equality (3.15) holds if the co-integration error u_t does not block Granger cause to the input change Δx_t in (3.4) (or $\Psi_{12i} = 0$ for any i=1,2,...,q).

Thus a sufficient condition in (3.15) that the TECHSHOCKTREND may affect the output in the long run is that the cointegration error affects the changes of inputs in the short run.

To test the null hypothesis in (3.15), we rewrite the equations for Δx_t in (3.4) as:

(3.16) $$\Delta x_{t+1} = \Psi_{01} + \sum_{i=1}^{q-1}\Psi_{11i}\Delta x_{t-i+1} + \sum_{i=1}^{q}\Psi_{12i}u_{t-i+1} + \delta_{t+1},$$
$$= \Psi_{01} + \sum_{i=1}^{q-1}\Psi_{11i}\Delta x_{t-i+1} + \Lambda u_t + \sum_{i=1}^{q-1}\Lambda_i\Delta u_{t-i+1} + \delta_{t+1}$$

where $\Lambda = \sum_{i=1}^{q}\Psi_{12i}$ and $\Lambda_i = -\sum_{j=1}^{q}\Psi_{12i+j}$. Therefore, a test of the null hypothesis $H_0 : \sum_{i=1}^{q}\Psi_{12i} = 0$ in (3.15) is equivalent to that $H_0 : \Lambda = 0$ in (3.16).

To construct a test statistic for this null hypothesis, define $B = (\underset{2\times1}{\Psi_0}\,\underset{2\times2}{\Psi_{111}},..., \underset{2\times2}{\Psi_{12q-1}}, \underset{2\times1}{\Lambda}, \underset{2\times1}{\Lambda_1},..., \underset{2\times1}{\Lambda_{q-1}})$, which is the component coefficient matrices in (3.16). Further, define a stacked variables $\Delta \mathrm{x}_{-i} = (\Delta x_{t-i}', \Delta x_{t-i-1}', \cdots, \Delta x_{t-i-n+1}')'$, $\hat{\mathrm{u}}_{-i} = (\hat{u}_{t-i}, \hat{u}_{t-i-1}, \cdots, \hat{u}_{t-i-n+1})'$ and $Z = (\mathrm{i}, \Delta\mathrm{x}_0, \Delta\mathrm{x}_{-1},...,, \Delta\,\mathrm{x}_{-q+2}, \hat{\mathrm{u}}_0, \Delta\hat{\mathrm{u}}_0\Delta\hat{\mathrm{u}}_{-1}\cdots, \Delta\hat{\mathrm{u}}_{-q+2})$ with $\underset{n\times1}{\mathrm{i}} \equiv (1,1,...,1)$. The OLS estimator of B' becomes $B' = (Z'Z)^{-1}Z'\Delta\mathrm{x}_{+1}$. Finally, the Wald test for $H_0 : \Lambda = 0$ may be written as (Lütkepohl , 1993; p93)

$$h_n \equiv n(C \times vec(\hat{B}))'[C(\hat{\Xi}^{-1} \otimes \hat{\Sigma})C']^{-1}C \times vec(\hat{B}),$$

where n is a sample number, $\hat{\Xi} = Z'Z/n$, $\hat{\Sigma} = (\Delta x_{+1} - \hat{B}Z')'(\Delta x_{+1} - \hat{B}Z')/n$ and $C = \begin{pmatrix} \underset{2\times[4(q-1)+2]}{0} & I_2 & \underset{2\times[2(q-1)]}{0} \end{pmatrix}$ is a selection matrix of Λ in $vec(B)$.

Theorem 3.5 *Suppose (i)* $H_0 : \Lambda = 0$ *hold, and* $p\lim\hat{\Xi}$ and $p\lim\hat{\Sigma}$ *are both non-singular. Then*

(a) $h_n \xrightarrow{d} \chi^2_{(2)}$.

(b) $\hat{\gamma}'[\hat{\theta}_{11}(1) + \hat{\theta}_{12}(1)\hat{\bar{\gamma}}']\sum_{s=1}^{t}\hat{\delta}_s \xrightarrow{p} \gamma'[\theta_{11}(1) + \theta_{12}(1)\bar{\gamma}']\sum_{s=1}^{t}\delta_s$

and $\hat{\gamma}'\hat{\theta}_{12}(1)\sum_{s=1}^{t}\hat{\tilde{\xi}}_s \xrightarrow{p} \gamma'\theta_{12}(1)\sum_{s=1}^{t}\tilde{\xi}_s$ *for any given t.*

See Kim (2014, 2016, 2017; Theorem 3) for the proof. Theorem 3. 5 holds mainly due to the super-consistency of the co-integration coefficient $\hat{\gamma}$.

□

4. Empirical Application for the United States Data

In this section, we conduct inferences for the trends in output using yearly data of the United States. The data source is FRED of Federal Reserve Bank of St. Louis. The labor is ‘average annual hours worked by persons engaged for United States (Not Seasonally Adjusted)’ and capital is ‘capital stock at constant national prices for United States (millions of 2011 U.S. Dollars, not seasonally adjusted)’. The output is ‘real gross domestic product (billions of chained 2009 dollars, not seasonally adjusted)’. The data period is from 1950 to 2014 that is all available in FRED. We demeaned and time-detrended the variables after log-transformation.

We then first conducted the Augmented Dickey-Fuller (ADF) and Elliott-Rothenberg-Stock point optimal tests for a unit root checking of these variables. We could not reject the null hypothesis that a variable has a unit root with a 1% level in every case. See Table 4.1. So we conclude that capital, labor and output are all I(1).

Table 4.1: Unit Root Test Results for Model Variables[1)]

variable	test type	include in test equation		
		none	intercept	trend and intercept
capital	ADF	0.448	0.978	0.016
	Elliott-Rothenberg-Stock point optimal [2)]	-	26.79	155.8
labor	ADF	0.027	0.208	0.500
	Elliott-Rothenberg-Stock point optimal	-	3.355	12.42
output	ADF	0.248	0.733	0.953
	Elliott-Rothenberg-Stock point optimal	-	10.06	28.71

Note: 1) P-value for null hypothesis: the variable has a unit root and a lag length is selected by the SIC.

2) Test critical values for 1% level are 1.99 (when an intercept is included in test equation) and 3.96 (when trend and intercept are included in test equation) according to Elliott-Rothenberg-Stock (1996, Table 1). The spectral estimation method is AR spectral OLS.

For the construction of a VAR model of capital, labor and output, the VAR lag order needs to be selected. See Table 4.2 for the computed five criterions LR (Log likelihood), FPE (Final prediction error), AIC (Akaike information criterion), SC (Schwarz information criterion) and HQ (Hannan-Quinn information criterion). Note the lags 2-5 has been selected by each criterion and thus we could not select a unfied order.

Table 4.2 Computed VAR Lag Order Selection Criteria[1)]

Lag	LR	FPE	AIC	SC	HQ
0	NA	6.77e-10	-12.60020	-12.49456	-12.55896
1	338.7444	1.94e-12	-18.45410	-18.03155*	-18.28916
2	25.79794	1.61e-12	-18.64513	-17.90567	-18.35648
3	24.84197	1.32e-12*	-18.84703*	-17.79065	-18.43466*
4	3.551771	1.68e-12	-18.61915	-17.24587	-18.08308
5	22.27884*	1.39e-12	-18.83218	-17.14198	-18.17240
6	6.616672	1.64e-12	-18.69251	-16.68540	-17.90902

Note: * indicates the lag order selected by the criterion.

We then tested whether the residuals of each VAR model is a white noise or not through the LM tests. See Table 4.3 for the results. In this test, VAR (6) has been selected that shows the highest p-values.

Table 4.3 VAR Residual Serial Correlation LM Tests[1) 2)]

Lags	VAR(1)	VAR(2)	VAR(3)	VAR(4)	VAR(5)	VAR(6) *
1	0.0005	0.1322	0.4503	0.2769	0.5476	0.7255
2	0.0001	0.0041	0.0946	0.0876	0.3546	0.2076
3	0.0235	0.4965	0.4594	0.0301	0.1991	0.9632

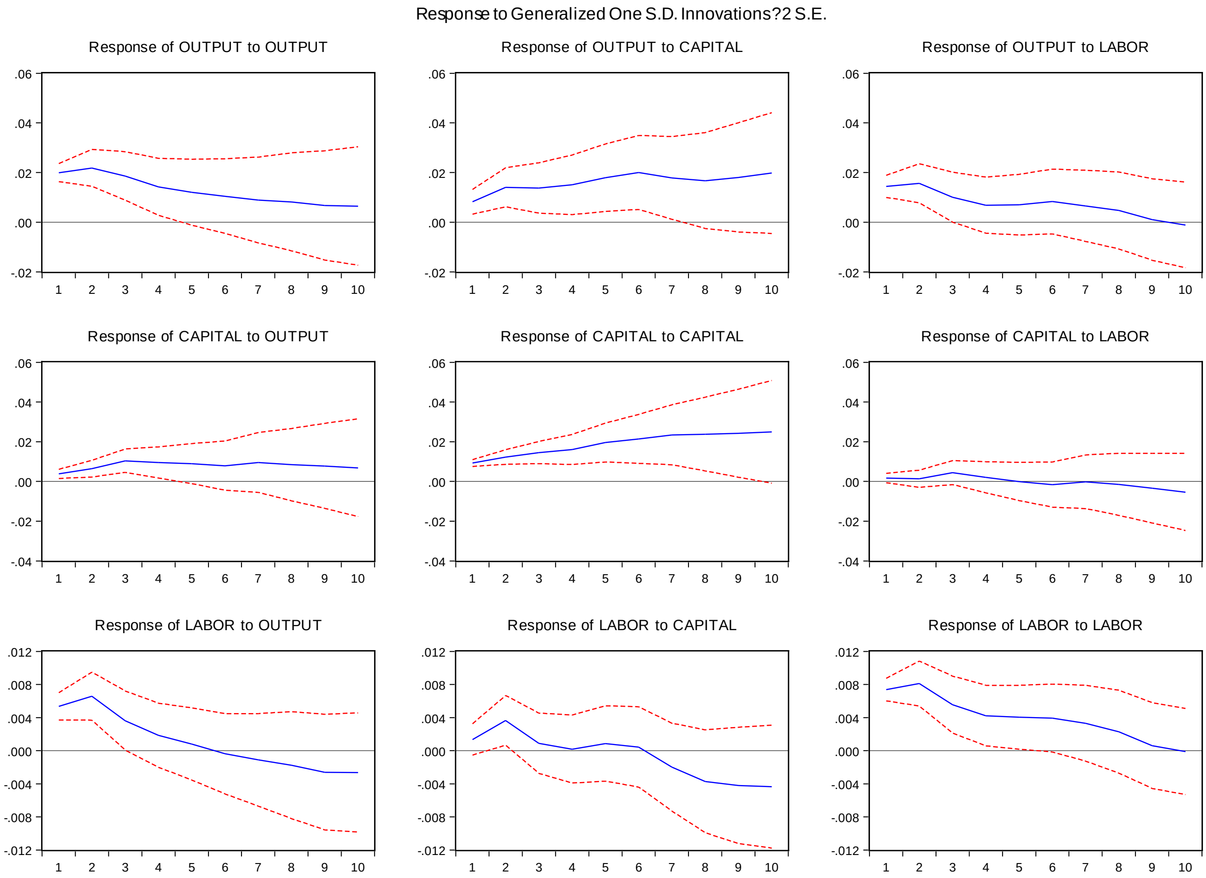
Note: 1) Probs from chi-square with 9 degree of freedom.
2) Null Hypothesis: no serial correlation at lag order h.

The above results of computed two criteria in Tables 4.2 and 4.3 were not coincided with each other and thus we selected VAR(6) that is the most general one among candidates. It is to escape from the omitted variable bias. [10]

We then conduct a generalized impulse response analysis to check whether capital or labor affects to the output significantly as the VAR-SGM implied. See Figure 4.4 for this analysis result using VAR(6) model of capital, labor and output. In there, we can see that the impulse of output induces a significant response of capital or labor during a future 2-3 years. Further the impulse of capital induces a significant response of capital or labor during a future 7 years that is the longer than the other two shocks labor or output.

[10] We also test whether the residuals of the VAR(6) model are multivariate normal and heteroskedastic. The null hypotheses that (i) the residuals are multivariate normal (Orthogonalization: Residual Correlation (Doornik-Hansen)) is rejected at the 1% significance level and (ii) there is no heteroskedasticity(no-cross terms) test is not rejected at the 1% significance level. However, non-normality and heteroskedasticity may not change the main results of this section. For the estimation and inference of the cointegration vector and space, Johansen's ML approach (especially the Bartlett-corrected trace statistic) show relatively robust and more competitive results than the other methods under non-normality and heteroskedasticity. See Gonzalo (1990) for this issue.

Figure 4.4: Impulse Response Analyses for VAR Model of Capital, Labor and Output



Note: standard error of response is computed by Monte Carlo simulation.

Now we conduct Johansen co-integration test to check whether there is a co-integration vector in the model. See Table 4.5 for the results. The null hypothesis that 'the hypothesized number of the co-integration is zero' was rejected in VAR(6) model at 5% level. So we may assume that there is a co-integration vector in the model.

Table 4.5: Johansen Unrestricted Co-integration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.[2)]
None [*1)]	0.235497	26.45946	24.27	0.0261[*]
At most 1	0.149152	10.88475	12.32	0.0860
At most 2	0.025807	1.516478	4.129	0.2558

Note: 1) Trace test indicates 1 cointegrating eqn(s) at the 0.05 level.
2) MacKinnon-Haug-Michelis (1999) p-values.

Following Table 4.6 shows that the estimators of cointegrating coefficients that is normalized by the coefficient of output. In there, we find that the output is positively related with the capital and labor in the long run. Note the labor has 1.5 times more weight than that of the capital.

Table 4.6: Estimator of Johansen Co-integration Coefficients

	$\ln Y_t$	$\ln K_t$	$\ln L_t$
Normalized estimated coefficients	1.000000	-1.901051 (0.37703)	-2.872176 (1.12210)

Note: 1) standard error in parentheses.

2) We assume that there is a co-integration vector.

For the robustness checking of the above Johansen co-integration test, we also conduct Engle-Granger test by following OLS estimation:

$$\ln Y_t = \underset{(0.048)}{0.696} * \ln K_t + \underset{(0.192)}{0.972} * \ln L_t \tag{4.1}$$

$$\textit{Adjusted } \mathrm{R}^2 = 0.765,$$

where a standard error is in parenthesis. Note the labor has 1.4 times more weight than that of the capital, which is close to the above Johansen estimation result. We then computed the Dickey-Fuller t-test statistic of the residual from above regression (4.1). We could reject the null hypothesis that ‘there is a co-integration’ with a 1% level in every case. See Table 4.7.

Table 4.7: Dickey-Fuller t-Test Statistic [1)]

variable	include in test equation		
	none	intercept	trend and intercept
Engle-Granger residual	-4.78 (-3.84)*	-4.75 (-4.31)	-4.80 (-4.36)

Note: 1) P-value for null hypothesis: the variable has a unit root and a lag length is selected by the SIC.

2) The number in parenthesis is Phillips and Ouliars (1990) 0.01 level critical value.

While above methods equally indicate the existence of cointegration, we will use the estimators of Johansen co-integration coefficients for the inference of model (rather than Engle-Granger OLS estimator). That is because stationary transformed error correction model (3.4) is derived under Assumption 2.2 on singular long run impact matrix which is supposed in Johansen estimation of co-integration vector (c.f. Kim; 2012).

Now we may test whether a time trend does not exist in the capital or labor(i.e., $\psi_{01} = 0$ in equation (3.5)) and the co-integration error is a mean zero process (i.e., $\psi_{02} = 0$ in equation (3.5)). We may test this assumption from equation (3.4) using a t-test statistic for the intercept term, which has a standard limit distribution because of super-consistency of co-integration vector (c.f., Kim; 2014). We found that the null can not be rejected at the 1% level. See Table 4.8 for the test and estimation results of the transformed error correction model (3.4).

Table 4.8: Estimation Results of Transformed Error Correction Model

Dependent Variable	Δ lnK		Δ lnL		U	
	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
Intercept	*-0.004*	*0.640*	*0.020*	*0.022*	*0.040*	*0.078*
Δ lnK(-1)	0.672	0.003	0.248	0.166	-1.410	0.004
Δ lnK(-2)	0.279	0.224	-0.542	0.007	0.728	0.149
Δ lnK(-3)	-0.142	0.558	0.050	0.804	0.103	0.846
Δ lnK(-4)	0.533	0.025	-0.037	0.849	-1.460	0.006
Δ lnK(-5)	-0.203	0.344	-0.453	0.014	0.669	0.157
Δ lnL(-1)	0.196	0.348	0.165	0.347	-0.650	0.158
Δ lnL(-2)	0.605	0.004	-0.330	0.053	-0.869	0.051
Δ lnL(-3)	-0.421	0.047	-0.249	0.156	0.546	0.232
Δ lnL(-4)	-0.046	0.826	0.072	0.684	-0.379	0.416
Δ lnL(-5)	-0.175	0.402	-0.355	0.046	0.577	0.209
U(-1)	0.250	0.024	0.001	0.992	0.439	0.067
U(-2)	-0.119	0.398	-0.137	0.249	0.567	0.071
U(-3)	-0.174	0.218	0.071	0.547	-0.143	0.641
U(-4)	0.075	0.597	-0.053	0.653	-0.160	0.608
U(-5)	-0.111	0.422	-0.091	0.434	0.434	0.157
U(-6)	0.113	0.306	0.207	0.029	-0.297	0.221
Adjusted R-squared	0.510		0.198		0.901	
Prob(F-statistic)	0		0		0	
Durbin-Watson stat	1.949		2.049		1.924	

Further, we conducted the h_n -test, as in Theorem 3.5, in order to check the existence of TECHSHOCKTREND in output. We computed that h_n = 4.37 and thus could not reject the null hypothesis that ‘there is not a TECHSHOCKTREND in the output’ at the 1% level because the test statistic's critical value is 9.21.[11]

Finally, we estimated the OUTPUTTREND suggested in (3.11) as;[12]

(4.2)

$$\begin{aligned}\text{OUTPUTTREND}_t &\equiv \lim_{j\to\infty} E_t \ln Y_{t+j} \\ &= 7.763 * \text{CAPITALSHOCKTREND}_t + \\ &\quad 2.217 * \text{LABORSHOCKTREND}_t + 0.103 * \text{TECHSHOCKTREND}_t\end{aligned}$$

where $\hat{\lambda} = (0.093, 8.193)'$. See Figure 4.9 and 4.10 for the graphs of trends where the capital, labor and technology trends separately to see which component derived the major recessions where

(4.3)

$$\text{CAPITALTREND}_t = 7.763 * \text{CAPITALSHOCKTREND}_t\,,$$
$$\text{LABORTREND}_t = 2.217 * \text{LABORSHOCKTREND}_t$$
$$\text{TECHTREND}_t = 0.103 * \text{TECHSHOCKTREND}_t\,.$$

Note all trends defined in (4.3) have the same unit with the output[13] and thus we can compare the contributions of these trends to the output. See Figure 4.10 for the graphs of these trends. It is noteworthy that output and capital trends rapidly dropped during early 1990's recession and GFC. However the TECHTREND little contributes to OUTPUTTREND while CAPITALTREND dominates the movement of OUTPUTTREND.

From the above graph of OUTPUTTREND, we can see that the GFC is not a historical recession but a mean (or trend) reversion procedure from unusually high level, which has been induced by the decrease of CAPITALTREND.[14]

Then our following question is which cause(s) pushed the capital to unusually high level during the pre-GFC period (2000-2007). A possible explanation is the historically low long run expectation of real interest rate (RELEX) during pre-GFC period, which probably boosted the investment and capital formation. Further note the low RELEX has been again induced by the low inflation shock trend. See Kim(2017; Figure 4 and 5) for this issue.

[11] Mankiw, Romer, and Weil (1992) also find that the estimated impact of saving and population growth on income is far larger than predicted by the SGM (it supports our result).
[12] Eviews 7 and Gauss 7 were used for the computation. The codes are available on request.
[13] It is because those are computed by the estimated coefficients multiplied by each shock trend.
[14] We just feel it seriously because pre-crisis boom was historically high and sustained exceptionally long time of almost 10 years.

Figure 4.9: Graphs Output Shock Trend and Output Gap

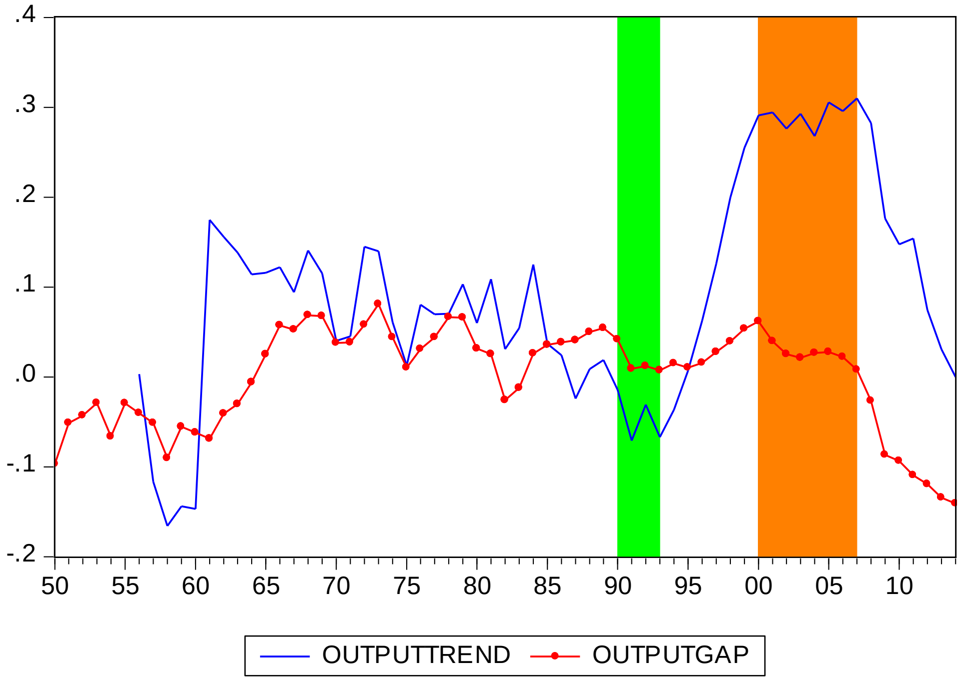
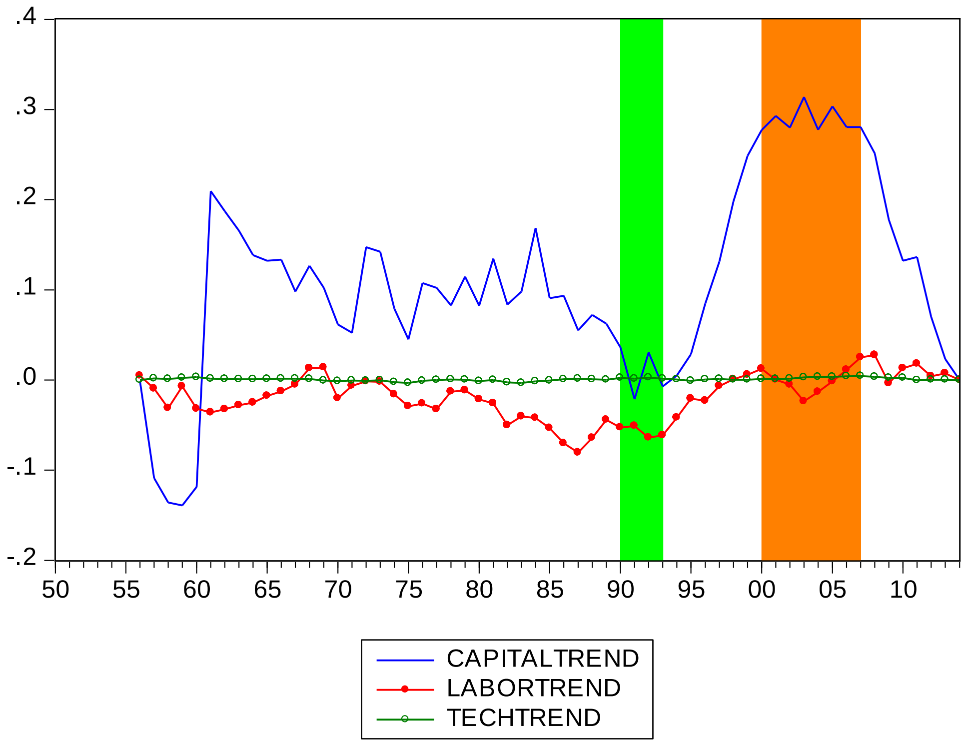


Figure 4.10: Graphs for TECHTREND, CAPITALTREND and LABORTREND



Further, for the comparison of OUTPUTTREND with the other measures of business cycles, we also graphed the output gap in Figure 4.9 that is defined as the difference between the actual and the potential real GDP, and the potential real GDP is estimated using a linear trend. See de Brouwer (1998) for the issues regarding the output gap estimation. Thus, the output gap is defined as a residual of the following regression:[15]

$$\ln Y_t = \underset{(0.0141)}{7.7540} + \underset{(0.0003)}{0.0318}\,t$$

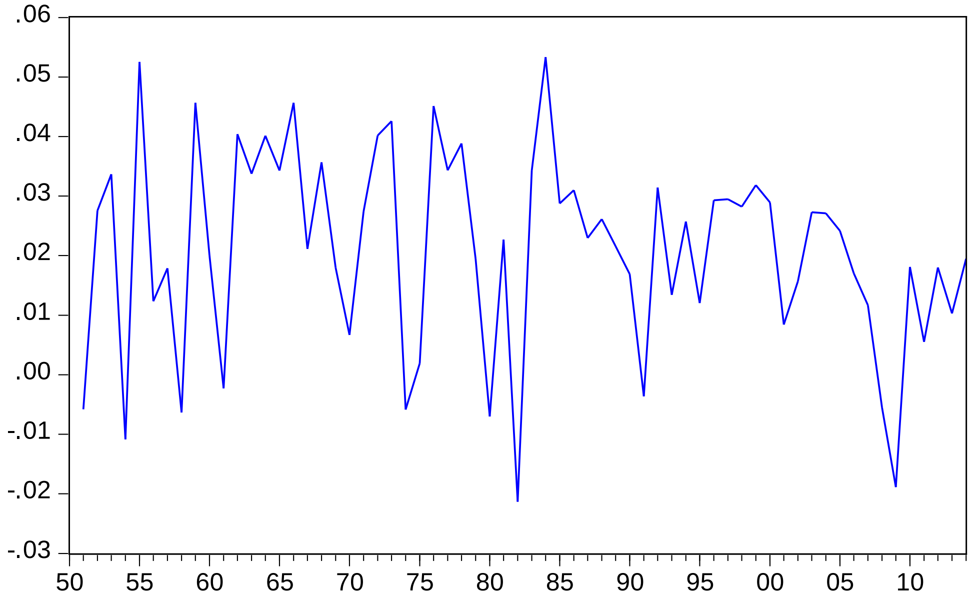
$$Adjusted\ R^2 = 0.991$$

where the number in parentheses is a standard error.

We also graphed the SR as in King and Rebelo (1999; 3-22) $\Delta \log SR_t = \Delta[\log Y_t - \alpha \log L_t - (1-\alpha)\log K_t]$ where $\alpha = 0.667$. See Figure 4.11 for the result.

Note the output gap showed a drop after the GFC while it was not historically increased right before the GFC. In this sense, suggested output gap does not provides much valuable information on the cause and characteristic of the GFC. The same critique is applied to the SR. There was sudden drop in Solow residual at the beginning of GFC and showed quick recovery from it. However that seems to be unrealistic because the economy has not recovered for a long time after GFC. Those are different from the indication of the output trend.

Figure 4.11: Solow Residual



See Table 4.12 for descriptive statistics of capital, labor and output shock trends. Note the CAPITALTREND has the largest standard deviation, which suggests that it dominates the variation of

[15] So the potential GDP is growing with rate 3.18%.

output.[16]

Table 4.12: Descriptive Statistics of CAPITALTREND, LABORTREND and TECHTREND

	CAPITALTREND	LABORTREND	TECHTREND
Mean	0.110	-0.019	0.001
Median	0.102	-0.016	0.001
Maximum	0.314	0.028	0.004
Minimum	-0.139	-0.080	-0.003
Std. Dev.	0.108	0.025	0.002
Observations	59	59	59

Then we conducted the test of the long run return to scale through a log likelihood (LR) test in Remark 3.1 where the estimate of $\gamma'(1,1)$ is 4.773. We found that IRS (increasing return to scale) has larger p values than the CRS and thus we conclude that the IRS holds in the long run in the US economy.[17] See Table 4.13 for the results.

Table 4.13: Tests of Co-integration Restrictions on $\gamma'(1,1)$

Hypothesized No. of CE(s)	Restricted Log-likehood	LR Statistic	Degrees of Freedom	Probability
$\gamma'(1,1)=1$	595.9	3.829	1	0.050
$\gamma'(1,1)=2$	596.8	1.898	1	0.168
$\gamma'(1,1)=3$	597.6	0.470	1	0.492
$\gamma'(1,1)=4$	597.8	0.049	1	0.823
$\gamma'(1,1)=5$	597.8	0.002	1	0.959

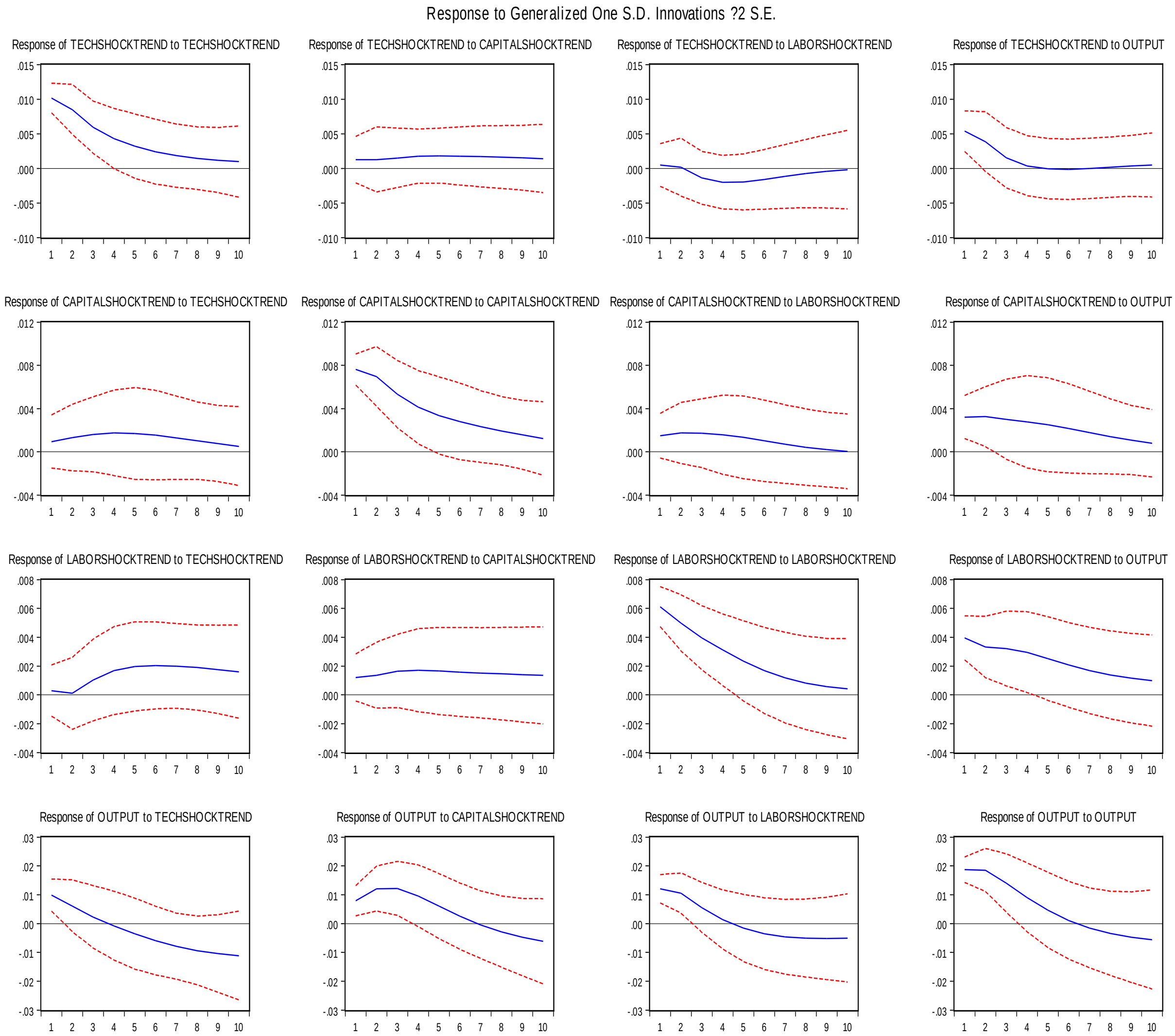
[16] Romer (2000; 187) "Real business cycle models posit technology shocks with a standard deviation about 1 percent each quarter. Yet it is usually difficult to identify specific innovations associated with the large quarter-to-quarter swings in the SR." See also Summers (1986) and Mankiw (1989).

[17] A SR is deduced under the CRS assumption. However if the IRS holds, then the SR may be affected by the capital or labor. For instance suppose $\log Y_t = \gamma_1 \log L_t + \gamma_2 \log K_t + u_t$ where $\gamma_1 + \gamma_2 > 1$. Then we may show $\log SR_t = (\alpha - \gamma_1)\log L_t + (1-\alpha-\gamma_2)\log K_t + u_t$. This derivation partially explain Romer (2000; 187) "More importantly, there is significance evidence that short-run variations in the SR reflect more than changes in the pace of technological innovation." See also Bernanke and Parkinson (1990), Mankiw (1989) and Hall (1988) for this issue.

Then we took dynamic analyses using a VAR model of the output and components of it (i.e., CAPITALSHOCKTREND, OUTPUTSHOCKTREND and TECHTSHOCKREND).[18] It is to check how these components affect the output dynamically.

For this, we first conducted the generalized impulse response analysis of Pesaran and Shin (1998) for the output, CAPITALSHOCKTREND, LABORSHOCKTREND and TECHSHOCKTREND.[19] See Figure 4.14 for the results. We can see that the shock of CAPITALSHOCKTREND induces longer (3 years) response of the output than LABORSHOCKTREND (2 years) or TECHSHOCKTREND (1 years).

Figure 4.14: Generalized Impulse Response Analysis Results



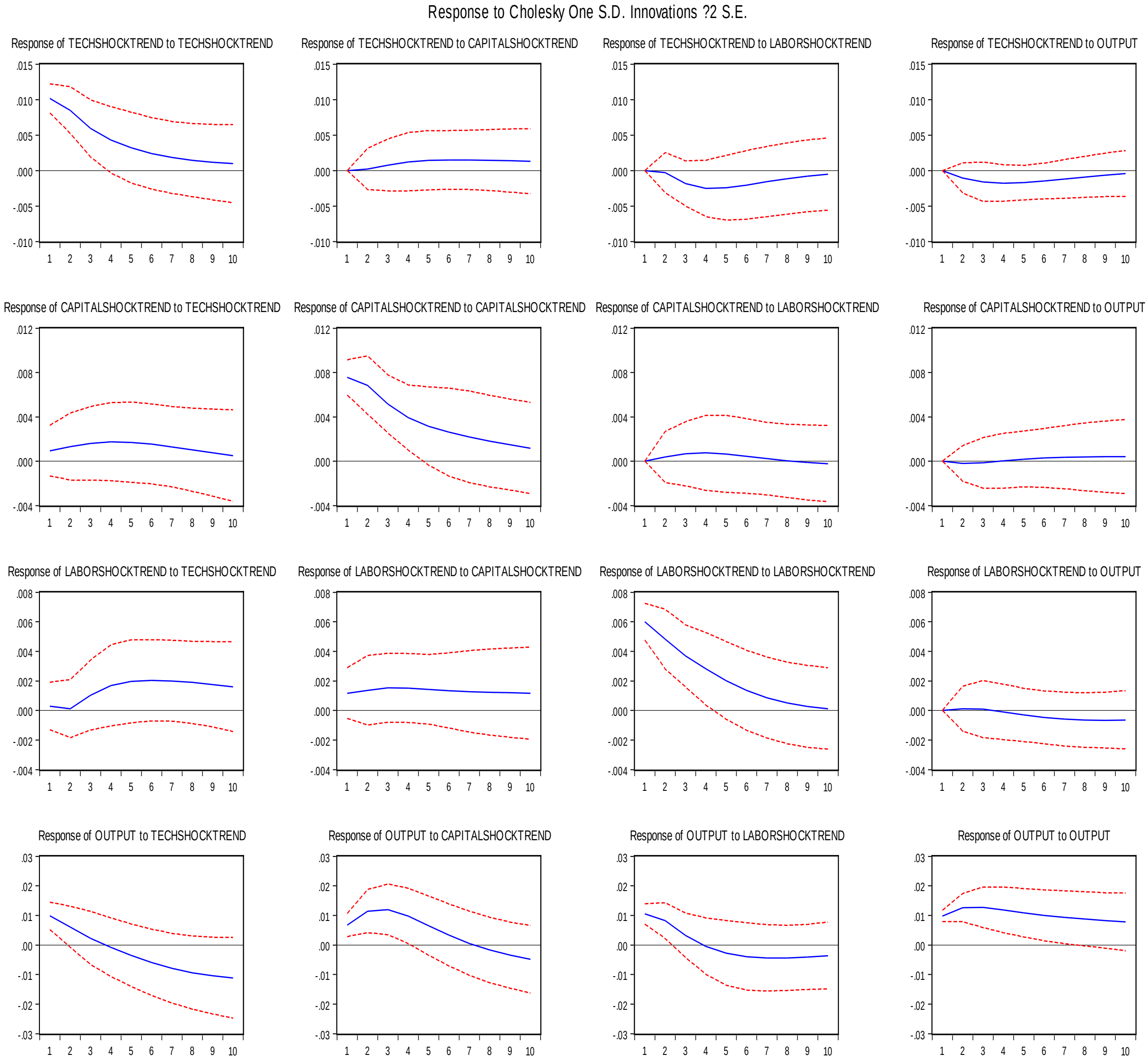
Note: Standard error of response is computed by Monte Carlo simulation.

[18] Remind that we could not reject the null hypothesis that 'there is not a TECHTREND in the output' at the 1% level. So we rather used TECHTSHOCKREND.

[19] We selected the VAR model lag as 2 following Schwarz criterion.

Then, for the robustness checking, we also conducted the impulse response analysis using Cholseky decomposition where the identification order is from the TECHSHOCKTREND, CAPITALSHOCKTREND, OUTPUTSHOCKTREND and output. We can see that the impulse response results are little changed by applying this alternative method except for the output shock. See Figure 4.15 for the results.

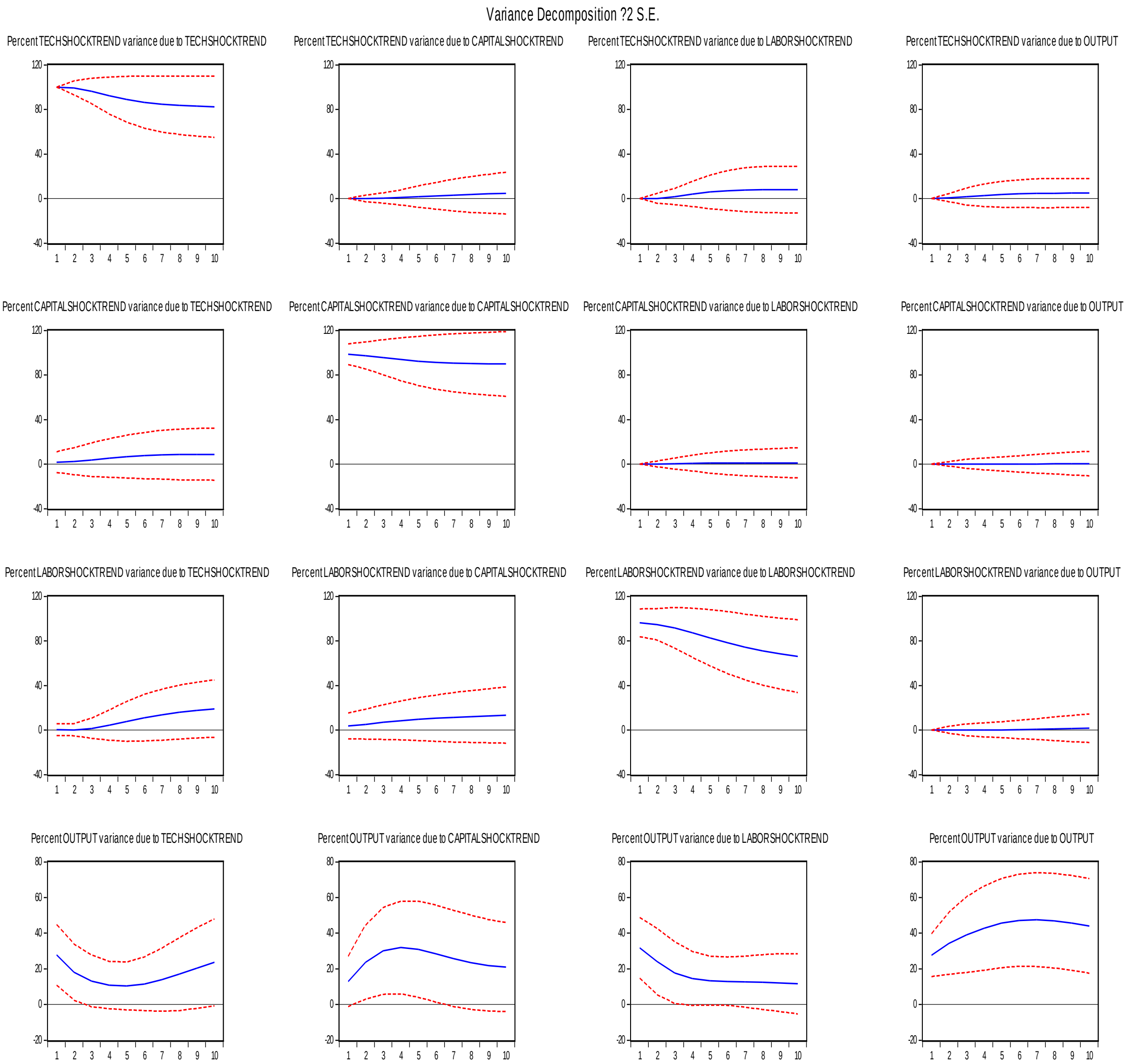
Figure 4.15: Ordinary Impulse Response Analysis Results



Finally, we also computed the variance decomposition, where the identification order is from the TECHSHOCKTREND, CAPITALSHOCKTREND, LABORSHOCKTREND and output. See Figure

4.16 for the results. For the case of the output variance decomposition, CAPITALSHOCKTREND has one year delayed and significant portion (20%-30%) during 2-6 years while LABORSHOCKTREND or TECHSHOCKTREND shows significant portion during less than 2 years.

Figure 4.16: Cholesky Decomposed Variance Decomposition



5. Conclusion

The capital and labor have been regarded as just exogenous inputs of an output production function in typical growth and business cycle models. However that convention might not be immune from Sims(1980)'s critique on the 'macroeconomics and reality'. It is because growth and business cycle models need to be obviously estimated and evaluated in a macroeconomic/econometric point even if its theoretical backgrounds partly depend on the microeconomics.

In this regard, we showed that a VAR generalization of Solow growth model may response to this necessity or critique. Using the US data, we found the sudden drops of capital shock trends may explain the causes of main business cycles; i.e., the early 1990's recession and the global financial crisis. Finally, we found that (i) we can not statistically reject the null hypothesis that a technology shock trend does not affect the output in the long run and (ii) the capital shock trend induces longer response of the output than labor or technology shock trends.

Based on the suggested empirical results on the importance of capital shock, it is surprising that the insight of Keynes (1936) is still valid after almost a century; i.e., classic housing or SOC construction (as a capital shock) is more important than Silicon Valley news on the R&D (as a technology shock) to understand the business cycles.

Remained work is to study how the capital shock is related with the fiscal or monetary policy as a major determinant of the output variation.

<Reference>

Baumol, W., 1986, Productivity Growth, Convergence, and Welfare: What the Long-run Data Show American Economic Review, vol. 76, issue 5, 1072-85.

Bernanke, B. and Parkinson, M., 1991, Procyclical Labor Productivity and Competing Theories of the Business Cycle: Some Evidence from Interwar U.S. Manufacturing Industries, Journal of Political Economy, University of Chicago Press, vol. 99(3), 439-459.

Beveridge, S. and C. Nelson, 1981, A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle, Journal of Monetary Economics 7(2), 151-174.

de Brouwer, G., 1998, Estimating Output Gaps, Discussion Paper No. 9809, Economic Research Department, Reserve Back of Australia.

DeLong, J., 1988, Productivity Growth, Convergence, and Welfare: Comment, American Economic Review 78: 5, 1138-1154.

Graham, E. Rothenberg, T. and J. Stock. Efficient Tests For An Autoregressive Unit Root, Econometrica, 1996, v64(4,Jul), 813-836.

Gonzalo, 1990, Five Alternative Methods of Estimating Long Run Equilibrium Relations, Journal of Econometrics, Volume 60, Issues 1–2, 203-233.

Hall, R., 1988, The Relation between Price and Marginal Cost in U.S. Industry, Journal of Political Economy, vol. 96(5), 921-947.

Hamilton, J., 1994, Time Series Analysis, Princeton University Press, Princeton, New Jersey.

Johansen, S., 1991, Estimation and Hypothesis Testing of Co-integration Vectors in Gaussian Vector Autoregressive Models," Econometrica 59(6), 1551-1580.

Keynes, 1936, The General Theory of Employment, Interest and Money, London: Macmillan.

Kim, Y-Y, 2017, Does Monetary Policy Affect Long Run Expectation of Non-Stationary Real Interest Rates ?." Applied Economics.

Kim, Y-Y, 2016, Dynamic Analyses for Transmission between Asset Bubble Trends of Accumulated Co-integration Errors, Asia Pacific Journal of Financial Studies, Vol. 45, No. 4, 574–605.

Kim, Y-Y, 2014, Inference for Stochastic Trend in Stock Price under Error Correction Model, Asia Pacific Journal of Financial Studies, Vol. 43, No. 3, 384-406.

Kim, Y-Y, 2012, Stationary Vector Autoregressive Representation of Error Correction Models, Theoretical Economics Letters, Vol. 2, No. 2, 152-156.

King, R., and Rebelo, S., 1999, Resuscitating real business cycles, Handbook of Macroeconomics, vol. 1, Part B, 927-1007, Elsevier.

Lütkepohl, H., 1993, Introduction to Multiple Time Series Analysis, Springer-Verlag.

Mankiw, G., 1989, Real Business Cycles: A New Keynesian Perspective. Journal of Economic Perspectives · vol. 3, no. 3, 79-90.

Mankiw, G., D. Romer, and D. Weil. 1992. A Contribution to the Empirics of Economic Growth". Quarterly Journal of Economics, 107(2), 407-437.

Ohanian, L., 2010, The Economic Crisis from a Neoclassical Perspective, Journal of Economic Perspectives, vol. 24. No. 4.

Pesaran M. H. and Y. C. Shin, 1996, Co-integration and Speed of Convergence to Equilibrium, Journal of Econometrics. Elsevier, 71 (1-2), 117-143.

Phillips, P. and Ouliaris, S., 1990, Asymptotic Properties of Residual Based Tests for Cointegration. Econometrica. 58 (1): 165–193.

Romer, David, 2000, Advanced Macroeconomics (2nd ed.). Boston: McGraw-Hill/Irwin.

Sims, C., “Macroeconomics and Reality,” Econometrica, Vol. 48, 1980, 1-48.

Solow R., 1957, Technical Change and the Aggregate Production Function, The Review of Economics and Statistics, Vol. 39, No. 3, 312-320.

Summers L., 1986, Some Skeptical Observations on Real Business Cycle Theory," Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 10, No. 4 , 23-27.

Watson, M. W., 1986, “Univariate Detrending Methods with Stochastic Trends,” Journal of Monetary Economics 18(1), 49-75.

May Monetary Policy Affect to Long Run Expectation of Non-Stationary Real Interest Rate ?

by

Yun-Yeong Kim

June, 2017

- Editorial Department -

531, College of Business &Economics,
152 Jukjeon-ro, Suji-gu, Yongin-si, Gyunggi-do, South Korea

(Post code) 16890 / (Tel) 82+ 31-8005-2660
(Fax) 82+31 8005-4033 / (Home) http://cms.dankook.ac.kr/web/rifi

The Research Institute of Future Industry
Dankook University

RIFI Working Paper 17-01