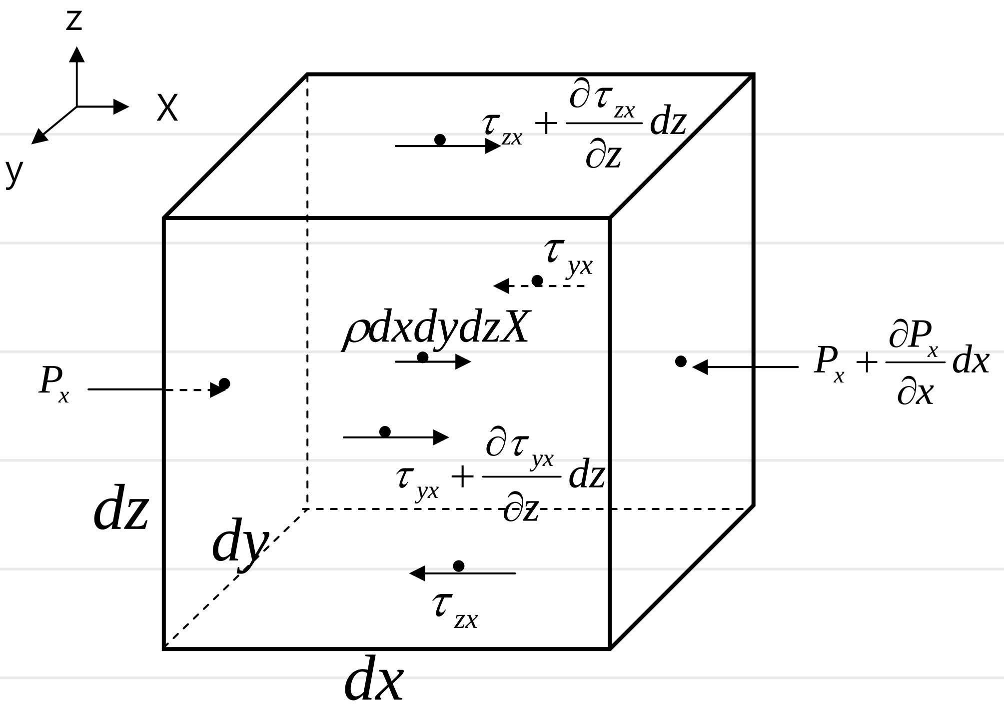
Navier-stokes 운동방정식



흐르는 유체속의 유체요소 dx, dy, dz에 작용하는 힘은

1. 6면체 질량에 작용하는 질량력
2. 6면체 표면에 작용하는 전단력
3. 6면체 표면에 작용하는 수직력

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x축 방향

질량(력): $\rho dxdydz$ 질량력: X (단위질량에 작용하는 힘)

전단력: $(-)\tau_{zx}, \quad \tau_{zx} + \dfrac{\partial \tau_{zx}}{\partial z} dz$

$$\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy$$

수직력: $P_x, \quad P_x + \dfrac{\partial P_x}{\partial x} dx$

$$\sum F_x = \rho dxdydz \frac{du}{dt} \rightarrow \rho \frac{du}{dt} = \rho X - \frac{\partial P_x}{\partial x} + \left(\frac{\partial \tau_{zx}}{\partial z} + \frac{\partial \tau_{yx}}{\partial y} \right)$$

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유체요소의변형과 전단력

$$\frac{\partial \tau_{zx}}{\partial x} = \frac{\partial}{\partial z}\left\{\mu\left(\frac{\partial u}{\partial z} + \frac{\partial \omega}{\partial x}\right)\right\} = \mu\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 \omega}{\partial x \partial z}\right)$$

$$\frac{\partial \tau_{yx}}{\partial y} = \frac{\partial}{\partial y}\left\{\mu\left(\frac{\partial \upsilon}{\partial x} + \frac{\partial u}{\partial y}\right)\right\} = \mu\left(\frac{\partial^2 \upsilon}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2}\right)$$

유체요소의변형과 수직력

$$\frac{\partial P_x}{\partial x} = \frac{\partial}{\partial x}\left(P - 2\mu\frac{\partial u}{\partial x}\right) = \frac{\partial P}{\partial x} - 2\mu\frac{\partial^2 u}{\partial x^2}$$

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$$\rho \frac{du}{dt} = \rho X - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

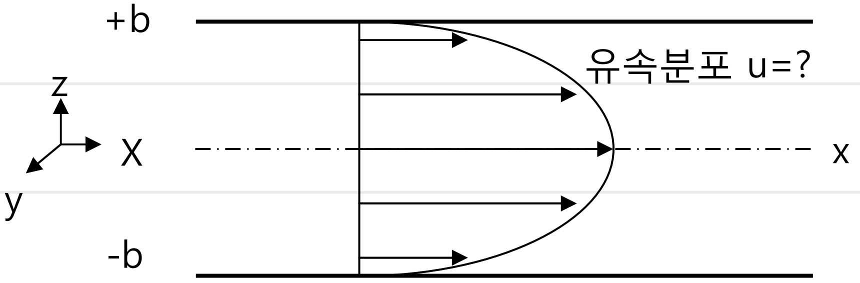
$$\rho \frac{d\upsilon}{dt} = \rho Y - \frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 \upsilon}{\partial x^2} + \frac{\partial^2 \upsilon}{\partial y^2} + \frac{\partial^2 \upsilon}{\partial z^2}\right)$$

$$\rho \frac{d\omega}{dt} = \rho Z - \frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2}\right)$$

Navier-stokes 운동방정식

예제 1) 평행한 두 평판사이의 2차원 점성류

x축을 수평이라 하고 정상류를 가정



$$X = 0, \quad \frac{du}{dt} = 0, \quad \frac{\partial u}{\partial x} = 0, \quad \frac{\partial^2 u}{\partial x^2} = 0, \quad \frac{\partial u}{\partial z} \neq 0$$

Navier – stokes **운동방정식**

$$\rho \frac{du}{dt} = \rho X - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho \frac{d\upsilon}{dt} = \rho Y - \frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 \upsilon}{\partial x^2} + \frac{\partial^2 \upsilon}{\partial y^2} + \frac{\partial^2 \upsilon}{\partial z^2}\right)$$

$$\rho \frac{d\omega}{dt} = \rho Z - \frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2}\right)$$

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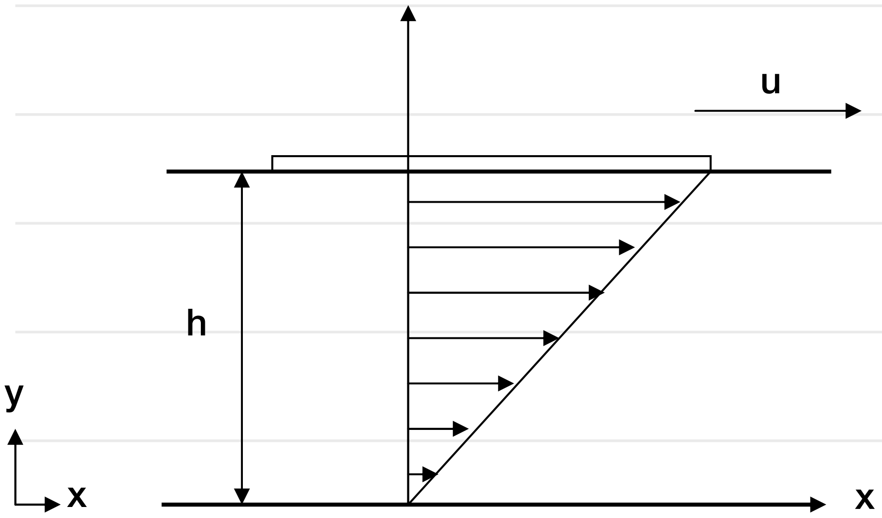
예제 2) **정상류,** $\frac{\partial P}{\partial x} = 0 \rightarrow x$**방향의 압력경사가 없다**

$y = 0$ **에서** $u = 0$

$y = h$ **에서** $u = U$



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예제 3) 점성 : μ, 층류 가정, 평균유속?